KEMENTERIAN PENDIDIKAN MALAYSIA

QS 015

Mid-Semester Examination

Semester I

Session 2013/2014

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1. Evaluate $\frac{1}{3-\sqrt{5}}$ - $\frac{1}{1+\sqrt{5}}$.

2. Solve the iequality $x - 1 < x^2 - 3 \leq 2x + 5$.

3. If $log_a \left(\frac{x}{a^2}\right) = 3\ log_a\ 2 - log_a(x - 2a)$, express x in terms of a.

4. (a) Express $z = -\sqrt{3} - i$ in polar form.

   (b) Given that the complex number z and its conjugate $\bar{z}$ satisfy the equation $z\bar{z} + 2iz = 12 + 6i$. Find the possible values of z.

5. (a) Given the first term of a geometric series is 40 and its sum to infinity is 60. Find the sum of the first forty terms, S_{40} of the series.

   (b) Expand $\frac{1}{(1-2x)^3}$ in ascending powers of x up to the term in x^3 and state the range of x such that the expansion is valid. Hence, approximate $(0.9)^{-3}$.

6. Given a matrix T = $\begin{bmatrix} 1 & 1 & p \\ 0 & -q & 1 \\ 3 & 2 & 1 \end{bmatrix}$.

   (a) Show that $|T| = 1 - q(1 - 3p)$.

   (b) Find the values of p and q if $|T| = 6$ and the minor for element a_{22} is -5.

   (c) Hence, determine the inverse of T, T^{-1} by using the adjoint method.

1. Evaluate $\frac{1}{3\text{-}\sqrt{5}}$ - $\frac{1}{1+\sqrt{5}}$.

Solution

$$\frac{1}{3\text{-}\sqrt{5}} - \frac{1}{1+\sqrt{5}}$$

$$= \frac{1}{3-\sqrt{5}}\left(\frac{3+\sqrt{5}}{3+\sqrt{5}}\right) - \frac{1}{1+\sqrt{5}}\left(\frac{1-\sqrt{5}}{1-\sqrt{5}}\right)$$

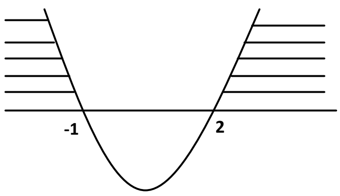
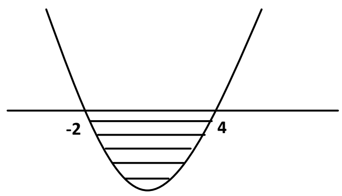
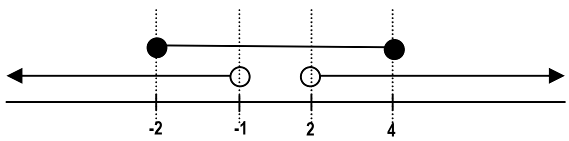
$$= \frac{3+\sqrt{5}}{4} + \frac{1-\sqrt{5}}{4}$$

$$= \frac{4}{4}$$

= 1

2. Solve the iequality $x - 1 < x^2 - 3 \le 2x + 5$.

Solution

<table>
<tr><td colspan="3">$x - 1 < x^2 - 3 \le 2x + 5$</td></tr>
<tr><td>$x^2 - 3 > x - 1$
$x^2 - x - 2 > 0$
$(x + 1)(x - 2) > 0$

$x < -1 \; or \; x > 2$</td><td>and</td><td>$x^2 - 3 \le 2x + 5$
$x^2 - 2x - 8 \le 0$
$(x + 2)(x - 4) \le 0$

$-2 \le x \le 4$</td></tr>
<tr><td colspan="3">
$\{x: -2 \le x < -1\} \cup \{x: 2 < x \le 4\}$</td></tr>
</table>

3. If $log_a\left(\frac{x}{a^2}\right) = 3\, log_a\, 2 - log_a(x-2a)$, express x in terms of a.

Solution

$$log_a\left(\frac{x}{a^2}\right) = 3\, log_a\, 2 - log_a(x-2a)$$

$$log_a\left(\frac{x}{a^2}\right) = log_a\, 2^3 - log_a(x-2a)$$

$$log_a\left(\frac{x}{a^2}\right) = log_a\left(\frac{8}{x-2a}\right)$$

$$\left(\frac{x}{a^2}\right) = \left(\frac{8}{x-2a}\right)$$

$$x^2 - 2ax = 8a^2$$

$$x^2 - 2ax - 8a^2 = 0$$

$$(x-4a)(x+2a) = 0$$

$$x = 4a \; or \; x = -2a$$

$$Since\; a > 0, \quad x \neq -2a$$

$$x = 4a$$

4. (a) Express $z = -\sqrt{3} - i$ in polar form.

(b) Given that the complex number z and its conjugate $\bar{z}$ satisfy the equation $z\bar{z} + 2iz = 12 + 6i$. Find the possible values of z.

Solution

(a) $r = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2} = 2$

$\propto = tan^{-1}\dfrac{1}{\sqrt{3}} = \dfrac{\pi}{6}$

$argument, \theta = -\dfrac{5\pi}{6}$

$Polar\ form,$

$$z = 2\left[cos\left(-\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)\right]$$

(b) $Let\ z = x + yi, \qquad \bar{z} = x - yi$

$(x + yi)(x - yi) + 2i(x + yi) = 12 + 6i$

$x^2 - y^2i^2 + 2xi + 2yi^2 = 12 + 6i$

$x^2 + y^2 - 2y + 2xi = 12 + 6i$

$x^2 + y^2 - 2y = 12$ ....(1)

$2x = 6$ ....(2)

$x = 3, \qquad 9 + y^2 - 2y = 12$

$y^2 - 2y - 3 = 0$

$(y - 3)(y + 1) = 0$

$y = 3, \quad y = -1$

$\therefore z = 3 + 3i, \quad z = 3 - i$

5. (a) Given the first term of a geometric series is 40 and its sum to infinity is 60. Find the sum of the first forty terms, S_{40} of the series.

(b) Expand $\frac{1}{(1-2x)^3}$ in ascending powers of x up to the term in x^3 and state the range of x such that the expansion is valid. Hence, approximate $(0.9)^{-3}$.

Solution

(a) $S_\infty = \frac{a}{1-r}$, $a = 40$

$$\frac{40}{1-r} = 60$$

$$\frac{40}{60} = 1 - r$$

$$\therefore r = \frac{1}{3}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{40} = \frac{40\left[1-\left(\frac{1}{3}\right)^{40}\right]}{1-\frac{1}{3}}$$

$$S_{40} = 60$$

(b) $(1-2x)^{-3}$

$$= 1 + (-3)(-2x) + \frac{(-3)(-4)}{2!}(-2x)^2 + \frac{(-3)(-4)(-5)}{3!}(-2x)^3 + \ldots$$

$$= 1 + 6x + 24x^2 + 80x^3 + \cdots$$

The expansion is valid when

$|2x| < 1$

$|x| < \frac{1}{2}$

$0.9 = 1 - 2x$

$\therefore x = 0.05$

$[0.9]^{-3} = [1 - 2(0.05)]^{-3}$

$= 1 + 6(0.05) + 24(0.05)^2 + 80(0.05)^3$

$= 1.37$

6. Given a matrix T = $\begin{bmatrix} 1 & 1 & p \\ 0 & -q & 1 \\ 3 & 2 & 1 \end{bmatrix}$.

(a) Show that $|T| = 1 - q(1 - 3p)$.

(b) Find the values of p and q if $|T| = 6$ and the minor for element a_{22} is -5.

(c) Hence, determine the inverse of T, T^{-1} by using the adjoint method.

Solution

(a) Expansion along first row

$$|T| = a_{11}c_{11} - a_{12}c_{12} + a_{13}c_{13}$$

$$= 1\begin{vmatrix} -q & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} + p\begin{vmatrix} 0 & -q \\ 3 & 2 \end{vmatrix}$$

$$= (-q - 2) - (0 - 3) + p(0 + 3q)$$

$$= -q - 2 + 3 + 3pq$$

$$= 1 - q\,(1 - 3p)$$

(b) Given

$$|T| = 6$$

$$1 - q(1 - 3p) = 6$$

$$-q + 3pq = 5 \quad \text{......................} \ (1)$$

$$Minor, a_{22} = -5$$

$$\begin{vmatrix} 1 & p \\ 3 & 1 \end{vmatrix} = -5$$

$$1 - 3p = -5$$

$$p = 2$$

$$-q + 3(2)q = 5$$

$$q = 1$$

$$\therefore p = 2, q = 1$$

(c) $adj\ T = C^T$

$$= \begin{bmatrix} +\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} \\ +\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 3 & 3 \\ 3 & -5 & 1 \\ 3 & -1 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 3 & 3 \\ 3 & -5 & -1 \\ 3 & 1 & -1 \end{bmatrix}$$

$Inverse\ of\ T,\qquad T^{-1} = \frac{1}{|T|}\ adj\ T$

$$= \frac{1}{6}\begin{bmatrix} -3 & 3 & 3 \\ 3 & -5 & -1 \\ 3 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{5}{6} & -\frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \end{bmatrix}$$