QS026/1
Mathematics
Paper 1
Semester II
Session 2005/2006
2 hours

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Matematik
Kertas 1
Semester II
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2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **9** halaman bercetak.
This booklet consists of **9** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of each of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

Coordinate Geometry

Perpendicular distance from the point (x_1, y_1) to the line $ax + bx + c = 0$ is

$$\mathrm{d} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Trapezium Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}, \text{ where } h = \frac{b-a}{n}$$

Newton-Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots$$

Sphere	$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$
Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	$S = \pi rs$
Right Circular Cylinder	$V = \pi r^2 h$	$S = 2\pi rh$

1. If $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are three nonzero vectors such that $\mathbf{u}+\mathbf{v}+\mathbf{w}=\mathbf{0}$, show that

$$\mathbf{u}\cdot\mathbf{v} = \frac{|\mathbf{w}|^2-|\mathbf{u}|^2-|\mathbf{v}|^2}{2}.$$

[5 marks]

2. Use the trapezoidal rule with four subintervals to approximate

$$\int_0^{\frac{\pi}{4}} \cos^4 x \, dx.$$

Give your answer correct to three decimal places. [6 marks]

3. Find an equation of the circle passing through the origin and its centre is the focus of the parabola $x^2 = 8y-16$. [6 marks]

4. Use Newton's method to find an approximate value of $\sqrt[3]{3}$. Give your answer correct to three decimal places. [7 marks]

5. The equation of a hyperbola is given by

$$y^2-4x^2-8x-4y-4=0.$$

Determine:

(a) the coordinates of its centre, foci and vertices. [7 marks]

(b) the equations of its asymptotes. [3 marks]

Sketch the graph of the hyperbola. [2 marks]

6. (a) Show that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\,\tan B}$. [5 marks]

(b) If β is an acute angle and $\beta=\sin^{-1}\dfrac{1}{\sqrt{5}}$, show that $\tan\beta=\dfrac{1}{2}$. Hence, by using the expansion of $\tan(A+B)$, show that

$$\tan^{-1}\left(\frac{1}{3}\right)+\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)=\frac{\pi}{4}.$$

[7 marks]

7. The motion of an object is governed by the equation

$$\frac{dv}{dt} = g - kv,$$

where v is the velocity at time t, g is the gravity and k is a constant.

(a) Find the velocity v by assuming that the object starts from rest. [10 marks]

(b) Deduce that after a long period of time, the object will move with a constant velocity $\frac{g}{k}$. [2 marks]

8. If $\tan\frac{x}{2} = t$, express $\sin x$ and $\cos x$ in terms of t. [2 marks]

Hence,

(a) find all values of t which satisfy $3\cos x - 4\sin x = 5$. [4 marks]

(b) evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin x}\, dx$. [6 marks]

9. The position vectors of the points P, Q and R are given respectively as

$$\mathbf{p} = 4\mathbf{i} + 3\mathbf{j} + 11\mathbf{k}, \quad \mathbf{q} = -2\mathbf{i} + 8\mathbf{k}, \quad \mathbf{r} = \mathbf{i} + 2\mathbf{j}.$$

(a) Show that PQR is a right angle triangle and calculate its area. [6 marks]

(b) Find an equation of the plane containing P, Q and R. [3 marks]

(c) Find parametric equations of the straight line passing through the point $(3, -5, 2)$ and perpendicular to the plane containing P, Q and R. [4 marks]

10. Given that $f(x) = \frac{4x}{1+x^2}$.

(a) Determine the intervals on which f is increasing and decreasing. Hence find the local extremum. [6 marks]

(b) Use the second derivative test to determine the intervals on which f is concave upward and concave downward. Hence, find the inflection points. [6 marks]

(c) Sketch the graph of f. [3 marks]

END OF QUESTION PAPER