

QS026/1
Mathematics
Paper 1
Semester II
2006/2007
2 hours

QS026/1
Matematik
Kertas 1
Semester II
2006/2007
2 jam



BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

CHOW CHOON WOOL

Kertas soalan ini mengandungi **13** halaman bercetak.
This booklet consists of 13 printed pages.

© Bahagian Matrikulasi

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}\end{aligned}$$

$$\begin{aligned}\sin A + \sin B &= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B &= 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A - \cos B &= -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}\end{aligned}$$

Limit

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{\sin h}{h} &= 1 \\ \lim_{h \rightarrow 0} \frac{1 - \cos h}{h} &= 0\end{aligned}$$

Hyperbolic

$$\begin{aligned}\sinh(x+y) &= \sinh x \cosh y + \cosh x \sinh y \\ \cosh(x+y) &= \cosh x \cosh y + \sinh x \sinh y \\ \cosh^2 x - \sinh^2 x &= 1 \\ 1 - \tanh^2 x &= \operatorname{sech}^2 x \\ \coth^2 x - 1 &= \operatorname{cosech}^2 x \\ \sinh 2x &= 2 \sinh x \cosh x \\ \cosh 2x &= \cosh^2 x + \sinh^2 x\end{aligned}$$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$\coth x$	$-\operatorname{cosech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int u dv = uv - \int v du$$

Sphere	$V = \frac{4}{3} \pi r^3$	$S = 4 \pi r^2$
Right Circular Cone	$V = \frac{1}{3} \pi r^2 h$	$S = \pi r s$
Right circular cylinder	$V = \pi r^2 h$	$S = 2 \pi r h$

1. Given that $\tan^{-1}(-2) = \alpha$ and $\tan^{-1}(3) = \beta$, find the value of $\cot(\alpha + \beta)$. [5 marks]

2. The vectors \mathbf{a} , \mathbf{b} and \mathbf{c} are such that $\mathbf{b} \times \mathbf{c} = 3\mathbf{i}$ and $\mathbf{c} \times \mathbf{a} = 2\mathbf{j} + \mathbf{k}$, where \mathbf{i} , \mathbf{j} and \mathbf{k} are unit vectors. Express $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} + \mathbf{b} + 4\mathbf{c})$ in terms of \mathbf{i} , \mathbf{j} and \mathbf{k} . [6 marks]

3. (a) Show that $\frac{d}{dx}(\cosh x) = \sinh x$. [2 marks]

(b) Evaluate $\int_0^1 x \sinh x^2 dx$ to three decimal places. [5 marks]

4. Milk is being poured into a hemispherical bowl of radius 4 cm at the rate of $3\pi \text{ cm}^3/\text{sec}$. If the depth of the milk in the bowl is h cm, its volume V is

$$V = \pi \left(4h^2 - \frac{h^3}{3} \right) \text{ cm}^3.$$

At the instant the milk is $\frac{3}{2}$ cm deep, find

(a) the rate of change of h . [4 marks]

(b) the rate of change of the radius of the milk's surface. [3 marks]

5. Show that $\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$.

Hence, evaluate $\int_0^{\frac{\pi}{6}} \cos 4x \sin 2x dx$, giving your answer in a fraction form.

[10 marks]

6. Let L be a line passing through the centre of the circle $x^2 + y^2 - 2x - 2y = 7$ and perpendicular to the line $3x + 4y = 7$. Find
- (a) the coordinates of the points of intersections of L and the circle. [9 marks]
- (b) the equations of the tangents to the circle parallel to $3x + 4y = 7$. [3 marks]
7. The conic section given by $9y^2 - 4x^2 + 18y + 16x - 43 = 0$ is a hyperbola.
- (a) Express the equation in the standard form. [3 marks]
- (b) Determine the coordinates of the centre, the vertices and the foci of the hyperbola. [4 marks]
- (c) Find the equations of the asymptotes. [2 marks]
- (d) Sketch the graph. [3 marks]
8. The points $A(-2, 1, 2)$, $B(5, -7, -3)$ and $C(3, 3, 1)$ lie on the plane Π_1 .
The equation of a second plane Π_2 , is given as $2x - y - 2z = 5$.
- (a) Find the vectors \overrightarrow{AB} and \overrightarrow{AC} . [3 marks]
- (b) Determine the Cartesian equation of Π_1 . [5 marks]
- (c) Find the acute angle between Π_1 and Π_2 , giving your answer in degrees. [5 marks]

9. The function f is defined by $f(x) = \frac{x^2 - 4x}{x^2 + 9}$.

- (a) Determine the asymptote(s) of f . [2 marks]
- (b) Find the critical number(s) of f and determine the intervals where f is increasing and f is decreasing. [6 marks]
- (c) Find the coordinates of the local extrema of f . [2 marks]
- (d) Sketch the graph of f . [3 marks]

10. (a) Show that $4 \sin 2\theta + 3 \cos 2\theta - 3 \sin \theta - 3 = \sin \theta(8 \cos \theta - 6 \sin \theta - 3)$. [3 marks]
- (b) Express $8 \cos \theta - 6 \sin \theta$ in the form of $R \sin(\theta - \alpha)$, where R is positive and α is an acute angle in radian. [5 marks]
- (c) By using the facts in (a) and (b), solve

$$4 \sin 2\theta = 3 \sin \theta - 3 \cos 2\theta + 3$$

for $0 \leq \theta \leq \pi$. Give your answers in radian correct to three significant figures.

[7 marks]

END OF QUESTION PAPER