QS026/1
Mathematics
Paper 1
Semester II
2007/2008
2 hours

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Matematik
Kertas 1
Semester II
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BAHAGIAN MATRIKULASI
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MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi 13 halaman bercetak.
This booklet consists of 13 printed pages.

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$
$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$
$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$
$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Limit

$$\lim_{h \to 0} \frac{\sin h}{h} = 1$$
$$\lim_{h \to 0} \frac{1 - \cos h}{h} = 0$$

Hyperbolic

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$
$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$
$$\sinh 2x = 2\sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$\coth x$	$-\operatorname{cosech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int u\,dv = uv - \int v\,du$$

Sphere	$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$
Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	$S = \pi r s$
Right circular cylinder	$V = \pi r^2 h$	$S = 2\pi r h$

1. The total cost of manufacturing k boxes of chocolates (a function of time, t) is given by

$$C(k) = 2k^2 + k + 900,$$

where $k(t) = t^2 + 100t$.

Compute the rate of change of the total cost with respect to time when $t = 1$.

[5 marks]

2. By using the identity $\sin^2 x + \cos^2 x = 1$, show that

$$\cos\left(\sin^{-1} x\right) = \sqrt{1 - x^2}.$$

Hence, compute $\cot\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$ without using calculator.

[6 marks]

3. Let $f(x) = \sinh x$. Prove that $f^{-1}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$.

[7 marks]

4. The position vectors **p**, **q**, **r** and **s** are given such that

$$(\mathbf{s} - \mathbf{p}) \cdot (\mathbf{q} - \mathbf{r}) = 0 \quad \text{and} \quad (\mathbf{s} - \mathbf{q}) \cdot (\mathbf{r} - \mathbf{p}) = 0.$$

(a) Show that $(\mathbf{s} - \mathbf{r}) \cdot (\mathbf{p} - \mathbf{q}) = 0$.

[4 marks]

(b) If $\mathbf{p} = 4\mathbf{i} + 5\mathbf{j}$, $\mathbf{q} = 3\mathbf{i} + 2\mathbf{j}$, $\mathbf{r} = -4\mathbf{i} + \mathbf{j}$ and $\mathbf{s} = x\mathbf{i} + y\mathbf{j}$, find the values of x and y.

[3 marks]

5. A straight line $2x + y = 4$ intersects a hyperbola $\frac{x^2}{4} - \frac{3y^2}{16} = 1$ at A and B.

(a) Find the coordinates of A and B.

[4 marks]

(b) Hence, find the equation of a parabola that passes through the points A, B and $(8, 0)$.

[6 marks]

6. Show that $\cos 6x = \cos 2x\left(4\cos^2 2x - 3\right)$.

Hence, evaluate

$$\int_0^{\frac{\pi}{6}} x\cos 2x\left(4\cos^2 2x - 3\right)dx$$

[11 marks]

7. Find the values of p, q and r which make the ellipse

$$4x^2 + y^2 + px + qy + r = 0$$

touches the x-axis at the origin and passes through the point $(1,2)$.
Express the equation obtained in the standard form and hence find its foci.

[13 marks]

8. Given that $f(x) = x^3 - 3x^2 - 9x + 11$.

(a) If f intersects the x-axis at $x = 1$, $x = p$ and $x = q$, find p and q.

[3 marks]

(b) Determine the intervals where f is increasing and f is decreasing.

[4 marks]

(c) Use the second derivative test to find the coordinates of the local extremum.

[4 marks]

(d) Sketch the graph of f.

[2 marks]

9. Given that $\mathbf{u} = 3\mathbf{i} + 3\mathbf{j} - a\mathbf{k}$ and $\mathbf{v} = b\mathbf{i} + 2\mathbf{k}$. If $\mathbf{u} \times \mathbf{v} = 6\mathbf{i} - 2\mathbf{j} + 12\mathbf{k}$, determine the values of a and b.

[4 marks]

Hence, determine

(a) the direction angles of $\mathbf{u}$.

[4 marks]

(b) the area of parallelogram with sides $\mathbf{u}$ and $\mathbf{v}$.

[2 marks]

(c) the angle between $\mathbf{u}$ and $\mathbf{v}$.

[3 marks]

10. (a) Let $P(x, y)$ be a point on a unit circle with centre O at the origin, such that OP makes an angle acute θ with the positive x-axis. Prove that

$$\sin^2\theta + \cos^2\theta = 1,$$

and hence, show that

$$\sec^2\theta = 1 + \tan^2\theta.$$

[5 marks]

(b) Show that the equation

$$\cos x(\sin x + \cos x) - 1 = 0$$

can be reduced to

$$\tan x(1 - \tan x) = 0.$$

Hence, solve for x on the interval $[0, 2\pi]$.

[5 marks]

(c) Find the area enclosed by the curve $f(x) = \tan x(1 - \tan x)$ and the x-axis in the first quadrant.

[5 marks]

END OF BOOKLET