

QS025/1
Mathematics
Paper 1
Semester II
Session 2011/2012
2 hours

QS025/1
Matematik
Kertas 1
Semester II
Sesi 2011/2012
2 jam



BAHAGIAN MATRIKULASI
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MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK

Kertas 1

2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.

This question paper consists of 15 printed pages.

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INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int u dv = uv - \int v du$$

| | | |
|--------------------------------|-----------------------------|-----------------------------|
| Sphere | $V = \frac{4}{3} \pi r^3$ | $S = 4 \pi r^2$ |
| Right Circular Cone | $V = \frac{1}{3} \pi r^2 h$ | $S = \pi r^2 + \pi r h$ |
| Right circular cylinder | $V = \pi r^2 h$ | $S = 2 \pi r^2 + 2 \pi r h$ |

LIST OF MATHEMATICAL FORMULAE

Numerical Methods

Iteration Method:

$$x_{n+1} = g(x_n), \quad n=1,2,3,\dots \text{ where } |g'(x_1)| < 1$$

Newton-Raphson Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n=1,2,3,\dots$$

Conics

Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$r = \sqrt{f^2 + g^2 - c}$$

$$d = \sqrt{a^2 + b^2 + 2ga + 2fb + c}$$

Parabola:

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

$$F(h+p, k) \text{ or } F(h, k+p)$$

Ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$F(h \pm c, k) \text{ or } F(h, k \pm c)$$

- 1 Show that $4x^2 - 16x + y^2 + 2y - 8 = 0$ is an equation of an ellipse with vertical major axis. Hence, find its centre and foci.

[5 marks]

- 2 Solve the differential equation $e^x \frac{dy}{dx} + 2e^x y = 1$, given that $y = 2$ when $x = 0$.

[6 marks]

- 3 By using the substitution $t = \sqrt{x}$, find the exact value of the integral

$$\int_4^9 \frac{\sqrt{x}}{x-1} dx.$$

[7 marks]

- 4 Show that the equation $(5 - 3x)^{\frac{1}{3}} = x$ has a root in the interval (1, 2). By using the Newton-Raphson method with the first approximation $x_1 = 1$, find an approximate root of the equation correct to three decimal places.

[7 marks]

- 5 A radioactive substance of mass N gram decays at the rate of $\frac{dN}{dt} = -kN$, where k is a constant. Initially the amount of the substance was 80 gram. After 100 years it decayed to 20 gram.

- (a) Express N in terms of the elapsed time t .

[5 marks]

- (b) Calculate the amount of the substance remains after 120 years.

[5 marks]

- 6 Find the values of A , B , C and D if

$$\frac{x^2 + 2x + 1}{x^2(x^2 - 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}.$$

Hence, evaluate $\int_2^4 \left(\frac{x^2 + 2x + 1}{x^2(x^2 - 1)} \right) dx$.

[12 marks]

- 7 Find the area of the region bounded by $y = \sin \sqrt{x}$, $x = 0$, $x = \pi^2$ and the x -axis.

If the region is rotated 360° about the x -axis, find the volume of the solid generated.

[12 marks]

- 8 Given the point $P(-4, 2, -3)$, the straight line $L: \frac{x+2}{4} = \frac{y}{-3} = \frac{z+1}{5}$ and the plane $\Pi: 2x + y + 2z = 9$. Find

- (a) an acute angle between the straight line L and the plane Π .

[3 marks]

- (b) an intersection point between the straight line L and the plane Π .

[5 marks]

- (c) a Cartesian equation of the plane containing the point P and the straight line L .

[5 marks]

- 9 P, Q and R are three points in space where $\overline{PQ} = \underline{a}$ and $\overline{PR} = \underline{b}$. Given

$$\underline{a} = 2\underline{i} + 2\underline{j} - \underline{k}$$

$$\underline{b} = \underline{i} + 2\underline{j} + 2\underline{k}$$

- (a) Find the area of the triangle PQR . [4 marks]
- (b) Find the parametric equations of the line L passing through the point $R(2, 0, 3)$ and parallel to vector \underline{a} . [4 marks]
- (c) If $\underline{u} = (|\underline{b}|\underline{a} + |\underline{a}|\underline{b})$ and $\underline{v} = (|\underline{a}|\underline{b} - |\underline{b}|\underline{a})$, evaluate $\underline{u} \cdot \underline{v}$. Hence, interpret the geometrical relationship between \underline{u} and \underline{v} . [5 marks]
- 10 A line segment joining $(-1, 0)$ and $(3, 4)$ is a diameter of a circle.

- (a) Find an equation of the circle. [3 marks]
- (b) Find an equation of the tangent to the circle at the point $(3, 4)$. [4 marks]
- (c) Find the points of intersection of the circle with its chord of which the midpoint is the origin. [8 marks]

END OF QUESTION PAPER