QS025/1
Mathematics
Paper 1
Semester II
Session 2013/2014
2 hours

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Matematik
Kertas 1
Semester II
Sesi 2013/2014
2 jam

KEMENTERIAN PENDIDIKAN MALAYSIA

BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **17** halaman bercetak.
This question paper consists of **17** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin\,(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos\,(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan\,(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\text{cosec} x) = -\text{cosec}\, x \cot x$$

$$\int f'(x) e^{f(x)}\, dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln \left| f(x) \right| + c$$

$$\int u\, dv = uv - \int v\, du$$

Sphere $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

Right circular cone $V = \frac{1}{3}\pi r^2 h$ $S = \pi r^2 + \pi r h$

Right circular cylinder $V = \pi r^2 h$ $S = 2\pi r^2 + 2\pi r h$

LIST OF MATHEMATICAL FORMULAE

Numerical Methods

Iteration Method:

$$x_{n+1} = g(x_n), \quad n = 1,2,3,\ldots \text{ where } \left|g'(x_1)\right| < 1$$

Newton-Raphson Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1,2,3,\ldots.$$

Conics

Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$r = \sqrt{f^2 + g^2 - c}$$

$$d = \sqrt{a^2 + b^2 + 2ga + 2fb + c}$$

Parabola:

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

$$F(h+p,k) \text{ or } F(h,k+p)$$

Ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$F(h \pm c,k) \text{ or } F(h,k \pm c)$$

1 Determine $\int \cot^2 2\theta \sin^3 2\theta \, d\theta$.

[5 *marks*]

2 Two circles of radius 5 units pass through the origin with their centres lie on the line $x + y = 1$. Show that the equations of the circles are

$$x^2 + y^2 + 6x - 8y = 0 \text{ and } x^2 + y^2 - 8x + 6y = 0.$$

[6 *marks*]

3 Given nonzero vectors $\underline{p}$ and $\underline{q}$ are perpendicular. Prove that

(a) $|\underline{p} + \underline{q}|^2 = |\underline{p}|^2 + |\underline{q}|^2$.

[3 *marks*]

(b) $|\underline{p} + \underline{q}| = |\underline{p} - \underline{q}|$.

[4 *marks*]

4 Given the points $A(1, 2, -2)$, $B(2, 4, 6)$ and $C(-4, 3, -1)$. Find the area of the triangle ABC.

[7 *marks*]

5 According to the Newton's law of cooling, the rate of change of temperature of an object is proportional to the difference in temperature between the object and the surrounding temperature, M. The law is given by the following differential equation

$$\frac{dT}{dt} = -k(T - M)$$

where $T(t)$ is the temperature of the object at time t and k is a constant.

(a) Express T in terms of time t.

[5 *marks*]

(b) A bowl of soup is removed from an oven with temperature at $60°$. What is the temperature after 5 minutes if $k = 0.04$ assuming that the temperature of the surrounding is $26.9°$.

[6 *marks*]

6 (a) Use trapezoidal rule with four subintervals to estimate $\int_0^2 x^2 e^x dx$, correct to four decimal places.

[6 *marks*]

(b) Given $f(x) = x^3 - 5x + 3$. Show that $f(x) = 0$ can be written as $x = g(x) = -\frac{3}{x^2 - 5}$. With the initial value $x_1 = 0.5$, find the roots of $f(x)$ by using iteration method. Hence, calculate the root of $f(x)$ accurate to three decimal places.

[6 *marks*]

7 Given $9x^2 - 72x + 16y^2 + 32y = -16$ is the equation of an ellipse.

(a) Write the equation in standard form.

[3 *marks*]

(b) Find the foci, vertices, lengths of the major and minor axes.

[6 *marks*]

(c) Sketch the graph.

[3 marks]

8 Express $\frac{7x^2 + 3x + 2}{(x+1)^2(x-2)}$ as partial fractions. Hence, evaluate $\int_3^4 \frac{7x^2 + 3x + 2}{(x+1)^2(x-2)}\,dx$.

[12 *marks*]

9 Given two planes $\pi_1 : x+2y+z=1$, $\pi_2 : 2x-y+4z=1$ and the straight line $L : \frac{x-2}{2} = \frac{y+3}{4} = \frac{z-1}{5}$.

(a) Find an acute angle between the planes π_1 and π_2.

[5 *marks*]

(b) Write the equation of L in parametric form. Hence, find the intersection point between the straight line L and the plane π_2.

[5 *marks*]

(c) Find a Cartesian equation of the plane which is orthogonal to the straight line L and passes through the point $(1, 2, -3)$.

[3 *marks*]

10 Given $f(x) = \ln x$.

(a) Sketch the graph of f. Shade the region R which is bounded by $f(x)$, x-axis, $x=1$ and $x=2$.

[3 *marks*]

(b) Find the area of R.

[5 *marks*]

(c) Find the volume of the solid generated when the region R is rotated 360° about the x-axis.

[7 *marks*]

END OF QUESTION PAPER