QM016/2
Matematik
Kertas 2
Semester I
Sesi 2003/2004
2 jam
Mathematics
Paper 2
Semester I
Session 2003/2004
2 hours



#### BAHAGIAN MATRIKULASI KEMENTERIAN PENDIDIKAN MALAYSIA

MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

#### PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI

SEMESTER EXAMINATION FOR MATRICULATION PROGRAMME

### SEMESTER I SESI 2003/2004

SEMESTER I SESSION 2003/2004

MATEMATIK
Kertas 2
2 jam
MATHEMATICS
Paper 2
2 hours

#### JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.

DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE INSTRUCTED.

**CHOW CHOON WOOL** 

Kertas soalan ini mengandungi 11 halaman bercetak. This question booklet consists of 11 printed pages.

#### INSTRUCTIONS TO CANDIDATE:

This question booklet consists of 10 questions.

Answer all questions.

The full marks are shown in the brackets at the end of each question or section.

All work must be clearly shown.

The usage of electronic calculator is allowed.

Numerical answers can be given in the form of  $\pi$ , e, surd, fractions or up to three significant figures, where appropriate, unless otherwise stated in the question.

# LIST OF MATHEMATICAL FORMULAE

# Differentiation

If 
$$y = g(t)$$
 and  $x = f(t)$ , then  $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ .

# Integration

$$\int u dv = uv - \int v du$$

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1. Given that f(x) = 2x + 1 and  $h(x) = 2x^2 + 4x + 1$ , find a function g such that  $(f \circ g)(x) = h(x)$ . [5]

Write g(x) in the form of  $a(x+b)^2 + c$ , where a, b and c are constants. [1]

2. Find the following limits, if they exist:

(a) 
$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$$
. [3]

(b) 
$$\lim_{x \to \infty} \frac{\sqrt{x} - 3}{x - 9}.$$
 [3]

3. If  $y = xe^{-x}$ ,

(a) find 
$$\frac{dy}{dx}$$
 and  $\frac{d^2y}{dx^2}$ . [4]

(b) show that 
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$
. [2]

4. Find the values of A, B and C which satisfy

$$\frac{x+2}{(1-x)(x^2+2)} = \frac{A}{1-x} + \frac{Bx+C}{x^2+2}.$$
 [4]

Hence, find 
$$\int \frac{x+2}{(1-x)(x^2+2)} dx.$$
 [3]

5. Given that 
$$x+t=xt$$
 and  $2ty-y^2=3$ , find  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$ . [4]

Hence, find the values of 
$$\frac{dy}{dx}$$
 when  $x = 2$ . [8]

[8]

6. Find a value of k so that the function

$$f(x) = \begin{cases} kx^2, & \text{if } x \leq 2. \\ 2x + k, & \text{if } x > 2. \end{cases}$$

is continuous. [4]

Hence, by using the definition of  $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ , determine whether f'(2) exists or not.

7. Parametric equations of a curve is given by

$$x = \frac{2t}{t^2 + 1}$$
 and  $y = \frac{3t^2 + 1}{t^2 + 1}$ .

Find

(a) 
$$\frac{dy}{dx}$$
 in terms of t. [6]

(b) 
$$\frac{d^2y}{dx^2} \text{ when } t = 1.$$
 [6]

8. Let  $f(x) = \begin{cases} 2^x, & \text{for } x > 0. \\ x^2 + 1, & \text{for } x \le 0. \end{cases}$ 

(a) Find 
$$\lim_{x\to 0} f(x)$$
. [4]

(b) Is the function 
$$f$$
 continuous at  $x = 0$ ? Give your reasons. [2]

(c) Sketch the graph of 
$$f$$
 from  $x = -2$  to  $x = 1$ . [2]

(d) Find the area of the region bounded by the curve 
$$f$$
,  $x$ -axis and the lines  $x = -2$ ,  $x = 1$ . [4]

9. Shade the region bounded by the curves  $y = \frac{1}{\sqrt{x}}$ , y = x and y = 2. [4] Find, in terms of  $\pi$ , the volume of the solid generated when the region is rotated through 360° about the x-axis. [8]

10. Given that f, g and h as follows:

$$f(x) = |x|,$$
  

$$g(x) = x^{2} - 1,$$
  

$$h(x) = \frac{1}{x}, x \neq 0.$$

- (a) Find  $F(x) = (f \circ g \circ h)(x)$ . State its domain and range. [4]
- (b) Find all the asymptotes of F and determine the interval where F is continuous. [2]
- (c) Find the values of x when F(x) = 5. [5]
- (d) Sketch the graph of F. [4]

**END OF QUESTION PAPER**