QM016/1
Matematik
Kertas 1
Semester I
Sesi 2003/2004
2 jam
Mathematics
Paper 1
Semester I
Session 2003/2004
2 hours

KEMENTERIAN PENDIDIKAN MALAYSIA

BAHAGIAN MATRIKULASI
KEMENTERIAN PENDIDIKAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
SEMESTER EXAMINATION FOR MATRICULATION PROGRAMME
SEMESTER I
SESI 2003/2004
SEMESTER I
SESSION 2003/2004

MATEMATIK
Kertas 1
2 jam
MATHEMATICS
Paper 1
2 hours

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE INSTRUCTED.

Kertas soalan ini mengandungi **11** halaman bercetak.
This question booklet consists of 11 printed pages.

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks are shown in the brackets at the end of each question or section.

All work must be clearly shown.

The usage of electronic calculator is allowed.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless otherwise stated in the question.

LIST OF MATHEMATICAL FORMULAE

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

For a geometric series

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$$

Binomial Expansion:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n \in N$ and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$.

1. By substituting $a = 3^x$, solve the equation

$$9^x + 3 = 28(3^{x-1}).$$ [5]

2. Find the sum of even numbers between 199 and 1999. [6]

3. Express $\dfrac{5x^2 + 17x + 17}{(x+2)(x+1)^2}$ as a sum of partial fractions. [7]

4. The sum of the first four terms of a geometric series with common ratio $-\dfrac{1}{2}$ is 30. Determine the tenth term and the infinite sum, S_∞. [7]

5. (a) Let matrix $A = \begin{bmatrix} 6 & -4 \\ 1 & 0 \end{bmatrix}$.

If $A^2 - pA - qI = 0$ where p and q are real numbers, I is a 2 x 2 identity matrix and 0 is a 2 x 2 null matrix, find p and q. [4]

(b) Given a matrix equation $AX = B$ as

$$\begin{bmatrix} 1 & -1 & -3 \\ 2 & -1 & -4 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}.$$

(i) Find the determinant of matrix A. [2]

(ii) Given the cofactor matrix of $A = \begin{bmatrix} 5 & p & 3 \\ -4 & 2 & -2 \\ q & -2 & 1 \end{bmatrix}$, find p and q. [2]

(iii) Determine the adjoint matrix of A and hence find the inverse of A. [2]

6. If $(x-1)$ and $(x+2)$ are factors of the expression $4x^4 - 6x^3 + ax^2 + bx - 12$, determine a and b. Hence, factorise the expression completely. [11]

7. Solve the following inequalities.

(a) $x^2 + x - 12 > 0$. [5]

(b) $\left|\dfrac{2x-1}{x+2}\right| > 1$. [8]

8. (a) Using the principle of mathematical induction, prove that $2 + 4 + 6 + ... + 2n = n^2 + n$, where n is a positive integer. [6]

(b) The sum of the first n terms of an arithmetic sequence is $\dfrac{n}{2}(4n + 20)$.

(i) Write down the expression for the sum of the first $(n-1)$ terms. [2]

(ii) Find the first term and the common difference of the above sequence. [5]

9 (a) Solve 3 ln 2x =3 + ln 27. [4]

(b) Given a complex number $z = \dfrac{i}{2-i}$

(i) State z in the form of $a + ib$ where a and b are real numbers. [2]

(ii) Find the modulus and argument of z. [3]

(c) Given the complex numbers u, v and w such that $\dfrac{1}{u} = \dfrac{1}{v} + \dfrac{1}{w}$. If $v = 1 - 3i$ and $w = 2 + i$, state u in the form of $a + bi$ where a and b are real numbers. [4]

10. (a) Matrices A and B are given as

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 & -1 & -4 \\ -1 & -1 & 3.5 \\ 1 & 1 & -1 \end{bmatrix}.$$

Find AB and hence find A^{-1}. [5]

(b) A company produces three grades of mangoes: X, Y and Z. The total profit from 1 kg grade X, 2 kg grade Y and 3 kg grade Z mangoes is RM20. The profit from 4 kg grade Z is equal to the profit from 1 kg grade X mangoes. The total profit from 2 kg grade Y and 2 kg grade Z mangoes is RM10.

(i) Obtain a system of linear equations to represent the given information. [3]

(ii) Write down the system in (i) as a matrix equation. [1]

(iii) Use the Cramer's rule to solve the system of linear equation. Hence, state the profit per kg for each grade. [6]

END OF QUESTION PAPER