QM016/2
Mathematics
Paper 2
Semester I
Session 2008/2009
2 hours

QM016/2
Matematik
Kertas 2
Semester I
Sesi 2008/2009
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks allocated for each question or section is shown in the bracket at the end of each question or section.

All steps must be shown clearly.

Only non-programmable scientific calculator can be used.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

5

Differentiation

If $y = g(t)$ and $x = f(t)$, then $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$$\frac{d^2 y}{dx^2} = \frac{\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)}{\dfrac{dx}{dt}}$$

Integration

$$\int u\, dv = uv - \int v\, du$$

1. Given $\ln y = e^{xy}$, find $\dfrac{dy}{dx}$.

[5 marks]

2. If $y = \sqrt{2x^2 + 5x - 3}$, determine the domain of $\dfrac{dy}{dx}$ and find the respective intervals

in which $\dfrac{dy}{dx} > 0$ and $\dfrac{dy}{dx} < 0$.

[6 marks]

3. Given that $f(x) = \dfrac{10 - 2x}{k}$ and $g(x) = 5 - 2x^2$. Find the value of k so that

$f^{-1}(x^2) = g\left(\dfrac{x}{2}\right)$. Hence, find $\left(f^{-1} \circ g\right)(0)$.

[7 marks]

4. Let $f(x) = |4x - 1|$ and $g(x) = x + 2$.

   (a) Find the interval of x for which $f(x) < g(x)$.

   [4 marks]

   (b) If $h(x) = f(x) + 2g(x)$, express $h(x)$ as a piecewise function.

   [3 marks]

5. Let $f(ax) = a^3 x^2 + a^2 x + 3a$ where a is non-zero.

   (a) Find a if $f(0) = 6$.

   [2 marks]

   (b) Determine $f(x)$.

   [3 marks]

   (c) Determine the domain and range of $f(x)$. Hence, state the
   interval in which f is one to one.

   [5 marks]

6. (a) By using the partial fraction method, show that

$$\frac{1}{x^2 - 4} = \frac{1}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right).$$

Hence, find $\int \frac{x^2 + 1}{x^2 - 4} \, dx$.

[6 marks]

(b) Sketch the region bounded by the curves $y = xe^{x^2}$, $y = x^2$, $x \geq 0$ and the line $x = 2$. Find its area.

[6 marks]

7. Given

$$f(x) = \begin{cases} e^x + A, & x < 0 \\ x^2 - 2x + 3, & 0 \leq x < 1 \\ x + B, & x \geq 1. \end{cases}$$

(a) Determine the values of A and B for f to be continuous.

[4 marks]

(b) Find the minimum value of f.

[3 marks]

(c) Is f differentiable? Justify your answer by using the first principle of differentiation.

[Hint: $e^x = 1 + x + \frac{x^2}{2!} + ...$]

[5 marks]

8. Given that

$$y = e^t + e^{-t} \quad \text{and} \quad x = e^{-t}.$$

(a) Find the point (x, y) on the curve where $\frac{dy}{dx} = 0$.

[6 marks]

(b) Solve for t if

$$\left(\frac{d^2 y}{dx^2}\right)^2 + \frac{dy}{dx} - 1 = 0.$$

[7 marks]

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9. Evaluate

(a) $\int \dfrac{1}{1+e^{-x}}\,dx$.

[7 marks]

(b) $\int \ln\left(x^{x}\right)dx$.

[6 marks]

10. Given $f(x) = \dfrac{x\left|x-1\right|}{(x-1)(x+2)}$.

(a) Show that f is equivalent to

$$g(x) = \begin{cases} \dfrac{x}{x+2}, & x>1 \\[2mm] -\dfrac{x}{x+2}, & x<1 \end{cases}$$

[3 marks]

(b) Determine the asymptotes and the points of discontinuity of g.

[6 marks]

(c) Sketch the graph of g.

[3 marks]

(d) Find the points of intersection of $g(x)$ with the straight line $y=x+2$.

[3 marks]

END OF BOOKLET