QS016/1
Mathematics
Paper 1
Semester I
Session 2010/2011
2 hours

QS016/1
Matematik
Kertas 1
Semester I
Sesi 2010/2011
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.
This booklet consists of **15** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

For a geometric series:

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}, \ r \neq 1$$

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n \in N$ and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \text{ for } |x| < 1$$

1 Dividing $M(x)=x^2+ax+b$ by $(x+1)$ and $(x-1)$ give a remainder of -12 and -16 respectively. Determine the values of a and b.

[6 *marks*]

2 Solve the equation

$$\ln x-\frac{3}{\ln x}=-2.$$

[6 *marks*]

3 The quadratic equation $x^2+3mx+2=0$ has roots α and β where m is a constant. Form a quadratic equation with roots $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ in terms of m.

[7 *marks*]

4 The sum S_n of the first n terms of an arithmetic progression is given by $S_n=pn+qn^2$. The sum of the first five and ten terms are 40 and 155 respectively.

(a) Find the values of p and q.

[3 *marks*]

(b) Hence, find the nth term of the arithmetic progression and the values of the first term, a and the common difference, d.

[4 *marks*]

5 Solve the following inequalities.

(a) $\dfrac{3x^2+x-4}{2x^2-3x-2} > 0.$

[4 *marks*]

(b) $\left|\dfrac{x-1}{x+3}\right| \leq 2.$

[8 *marks*]

6 (a) Given two complex numbers $z_1 = 2+i$ and $z_2 = 1-2i$.

(i) Express $z_1^{\,2} + \dfrac{1}{\bar{z}_2}$ in the form $x+yi$, where x and y are real numbers and $\bar{z}_2$ is the conjugate of z_2.

[4 *marks*]

(ii) Hence, find the modulus of $z_1^{\,2} + \dfrac{1}{\bar{z}_2}$.

[2 *marks*]

(b) Find the square roots of $-3+4i$.

[6 *marks*]

7 The following table shows the price (RM) per type of 0.5 kg cakes sold at the shops P, Q and R together with the total expenditure if a customer buys a number of each type of cake from the listed shops.

Cake Types / Shops	Banana	Chocolate	Vanilla	Total Expenditure (RM)
P	5	8	5	36
Q	4	6	6	30
R	5	9	7	40

Let the number of banana, chocolate and vanilla cakes bought from each shop be x, y and z respectively.

(a) Write the matrix equation $AX = B$ using the above information.

[1 *mark*]

(b) Obtain the adjoint matrix of A. Hence, find the inverse of matrix A.

[8 *marks*]

(c) Determine the values of x, y and z using the inverse matrix of A obtained in (b).

[2 *marks*]

8 A polynomial $f(x) = px^3 + (p+q)x^2 + (p+2q)x + 1$ has a factor $(x+1)$.

(a) Express q in terms of p.

[3 *marks*]

(b) Write $f(x)$ in terms of p and x. Determine the quotient when $f(x)$ is divided by $(x+1)$.

[3 *marks*]

(c) Hence, find the value of p if $x = 3$ is one of the roots for $f(x) = 0$. Using the value of p, factorize $f(x)$ completely.

[5 *marks*]

9 (a) Given that $\frac{1}{u} = 0.015151515... = p + q + s + ...,$ where p, q and s are the first three terms of geometric progression. If $p = 0.015,$ state the value of q and s in decimal form. Hence, find the value of u.

[4 *marks*]

(b) Find the expansion for $\left(1 - \frac{x}{16}\right)^{\frac{1}{3}}$ up to the term x^2. State the range of x for which the expansion is valid. Show that $\sqrt[3]{8 - \frac{x}{2}} = 2\left(1 - \frac{x}{16}\right)^{\frac{1}{3}}$.

Hence, by substituting $x = 2$, approximate $\sqrt[3]{7}$ correct to four significant figures.

[9 *marks*]

10 The graph of a quadratic function $y = ax^2 + bx + c$, where a, b and c are constants passes through the points $(-2, -10)$, $(1, 8)$ and $(2, 6)$.

(a) Obtain a system of linear equations to represent the given information.

[2 *marks*]

(b) Write the system of linear equations in the form of a matrix equation $AX = B$, where

$$X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

[2 *marks*]

(c) Find the determinant of the matrix A.

[2 *marks*]

(d) By using the Cramer's Rule, solve the matrix equation.

[7 *marks*]

(e) Hence, write the quadratic function of the graph and determine whether the graph has a maximum or minimum value.

[2 *marks*]

END OF QUESTION BOOKLET