QS015/1
Mathematics
Paper 1
Semester I
Session 2011/2012
2 hours

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Matematik
Kertas 1
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2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Arithmetic series:

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Geometric series:

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$$

Sum to infinity:

$$S_\infty = \frac{a}{1-r}, |r| < 1$$

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n \in N$ and $\binom{n}{r} = \frac{n!}{(n-r)!\, r!}$

$$(1+ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$$

$|ax| < 1$ where $n \in Z^-$ or $n \in \mathbb{Q}$

1   Solve the equation $3^{2x+1} - 28(3^x) + 9 = 0$.

[6 *marks*]

2   The functions f and g are defined as:

$$f(x) = \sqrt{x-1}, \quad x \geq 1$$
$$g(x) = x^2, \quad x \geq 0.$$

Find the inverse function, $f^{-1}(x)$ and determine its range. Then, evaluate $(f \circ g)(-2)$.

[6 *marks*]

3   The ninth term and the sum of the first fifteen terms of an arithmetic progression are 24 and 330 respectively. Find the first term, a and the common difference, d. Hence, find the least possible value n, such that the sum of the first n terms is greater than 500.

[6 *marks*]

4   Matrix A is given as $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -3 \\ 2 & 2 & -1 \end{bmatrix}$.

(a)   Given the cofactor matrix of A is $\begin{bmatrix} 3 & x+y & -2 \\ 0 & 1 & 2 \\ -3 & x^2 & -1 \end{bmatrix}$ where $x > 0$.

Determine the values of x and y.

[3 *marks*]

(b)   Given $A^2 - 4A + I = 0$, show that $A^3 = 15A - 4I$ where I is the 3×3 identity matrix. Hence, find A^3.

[4 *marks*]

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5   Given two complex numbers $z_1 = 5 + 3i$ and $z_2 = 2 - i$.

(a)   State $\overline{z_1}$ and $\overline{z_2}$.

[1 *mark*]

(b)   Determine the value of k if $\dfrac{1}{z_1} = k\overline{z_1}$.

[3 *marks*]

(c)   Find $z_1 z_2$. Hence, show that $\overline{z_1}\ \overline{z_2} = \overline{z_1 z_2}$.

[6 *marks*]

6   (a)   Given $f(x) = e^{-x}$ and $g(x) = x^2$.

(i)   Find the domain and range of f and g.

[2 *marks*]

(ii)   Show that $(g \circ f)(x) = e^{-2x}$.

[2 *marks*]

(b)   Given

$$h(x) = \begin{cases} e^{-2x}, & -\infty < x \le 0 \\ x+1, & x \ge 0. \end{cases}$$

(i)   Find $h^{-1}(x)$.

[5 *marks*]

(ii)   Sketch the graph for $h(x)$ and $h^{-1}(x)$.

[4 *marks*]

7   (a)   Solve the equation $\log(x-4)+2\log 3 = 1+\log\left(\dfrac{x}{2}\right)$.

[5 *marks*]

(b)   Find the solution set of the inequality

$$\left|\frac{x-3}{x+1}\right| < 2.$$

[7 *marks*]

8   (a)   Given that the sum of the first n terms, S_n of a series as $S_n = 1-\left(\dfrac{1}{3}\right)^n$.

Find an expression for the nth term. Show that the series is a geometric series and find the sum to infinity, S_∞.

[6 *marks*]

(b)   Expand $\left(1+\dfrac{2}{x}\right)^{\frac{1}{2}}$ in the ascending powers of x up to the term in x^3.

Hence, by substituting $x=3$, evaluate $\sqrt{\dfrac{5}{3}}$ correct to three decimal places.

[6 *marks*]

9   (a)   A function $f(x)$ is defined by $f(x) = \dfrac{3x}{x-6}$ for $x \neq 6$.

Show that $f(x)$ is a one-to-one function.

Find the values of x such that $(f \circ f)(x) = 0$.

[7 *marks*]

(b)   Given $f(x) = \sqrt{1-3x}$ and $g(x) = \dfrac{x}{2} - 1$.

Find $f\left(g^{-1}\left(-\dfrac{7}{2} \right) \right)$.

[6 *marks*]

10   The following table shows the quantities (unit) and the amount paid (RM) for pens bought from three shops.

Pen Shop	Pilot (unit)	Kilometrico (unit)	Papermate (unit)	Amount paid (RM)
S	1	p	$2p$	18.00
T	1	q	$3q$	31.00
U	1	r	$4r$	37.00

Given the price in RM per unit of pilot, kilometrico and papermate pens be x, y and z respectively.

(a)   Obtain a system of linear equations to represent the given information.

[1 *mark*]

(b)   Write the system in the form of a matrix equation $AX = B$ where

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

[1 *mark*]

(c)   Given the minor a_{11}, a_{21} and a_{22} of matrix A is 9, 12 and 8 respectively. Find the values of p, q and r.

[4 *marks*]

(d)   Find the determinant, cofactor, adjoint and A^{-1} of matrix A. Hence, find the values of x, y and z.

[9 *marks*]

END OF QUESTION PAPER