SM015/1
PSPM 1

2019/2020

14 OCTOBER 2019

KOLEJ MATRIKULASI KEDAH
Authored by: CHOW CHOON WOOI

KEMENTERIAN PENDIDIKAN MALAYSIA

Questions

SECTION A [45 marks]

This section consists of 5 questions. Answer all questions.

1. Given the complex numbers $z_1 = -i$ and $z_2 = 2 + i\sqrt{3}$.

   a. Express ${z_1}^2$ and $\overline{z_2}$ in the form $a + bi$, where $a, b \in \mathcal{R}$. **[2 marks]**

   b. From part 1(a), find $W = \frac{{z_1}^2 + \overline{z_2}}{z_1}$. Hence, find $|W|$ and argument W. **[7 marks]**

2. Solve the following:

   a. $3(5^{2x}) + 25^{\frac{1}{2}x+1} = 200$ **[5 marks]**

   b. $x + 4 \leq x^2 + x < 12$ **[5 marks]**

3. The sum of the first n terms of a sequence is given by $S_n = 2 + 3^{-4n}$.

   a. Find the value of constant c such that the n-th term is $c3^{-4n}$. **[3 marks]**

   b. Show that the3 sequence is a geometric series. **[4 marks]**

   c. Find the sum of the infinite series, S_∞. **[2 marks]**

4. Given matrix $A = \begin{bmatrix} 2 & 3 & 0 \\ -5 & 0 & 4 \\ 0 & 2 & 1 \end{bmatrix}$.

   a. Find the determinant of matrix A by expanding the first row. **[2 marks]**

   b. Calculate the adjoin of matrix A. Hence, find A^{-1}. **[5 marks]**

   c. Solve the equation $AX = B$, where $B = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, by using the answer obtained in part 4(b). **[2 marks]**

5. Given $f(x) = \frac{1}{1+\frac{1}{1+\frac{1}{x}}}$.

   a. Simplify $f(x)$ and evaluate $f\left(\frac{1}{2}\right)$. **[4 marks]**

   b. The domain of $f(x)$ is a set of real number except three numbers. Determine the numbers. **[4 marks]**

SECTION B [25 marks]

1. Solve the following:

   a. $log_2 2x = 2\ log_4(x+4)$ **[6 marks]**

   b. $2\left|\frac{x-3}{2x-1}\right| \geq 1$ **[7 marks]**

2. Given a function $f(x) = \ln(2x+1)$

   a. State the domain and range of $f(x)$. **[2 marks]**

   b. Find the inverse function of $f(x)$ and state its domain and range. Hence, find the value of x for which $f^{-1}(x) = 0$. **[7 marks]**

   c. Sketch the graph of $f(x)$ and $f^{-1}(x)$ on the same coordinate axes. **[3 marks]**

END OF QUESTION PAPER

Question A1

1. Given the complex numbers $z_1 = -i$ and $z_2 = 2 + i\sqrt{3}$.
   a. Express ${z_1}^2$ and $\overline{z_2}$ in the form $a + bi$, where $a, b \in \mathcal{R}$.
   b. From part 1(a), find $W = \frac{{z_1}^2 + \overline{z_2}}{z_1}$. Hence, find $|W|$ and argument W.

SOLUTION

a) $z_1 = -i$

$$z_2 = 2 + i\sqrt{3}$$

$${z_1}^2 = (-i)^2$$

$$= -1 + 0i$$

$$\overline{z_2} = 2 - i\sqrt{3}$$

b) $W = \frac{{z_1}^2 + \overline{z_2}}{z_1}$

$$= \frac{(-1) + (2 - i\sqrt{3})}{(-i)}$$

$$= \frac{1 - i\sqrt{3}}{(-i)}$$

$$= \frac{(1 - i\sqrt{3})(i)}{(-i)(i)}$$

$$= \frac{i - i^2\sqrt{3}}{-i^2}$$

$$= \frac{i + \sqrt{3}}{1}$$

$$= \sqrt{3} + i$$

$$|W| = \sqrt{\sqrt{3}^2 + 1^2}$$

$$= \sqrt{3+1}$$

$$= 2$$

$$Arg\, W = tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

Question A2

2. Solve the following:

   a. $3(5^{2x}) + 25^{\frac{1}{2}x+1} = 200$

   b. $x + 4 \leq x^2 + x < 12$

SOLUTION

a) $3(5^{2x}) + 25^{\frac{1}{2}x+1} = 200$

$$3(5^x)^2 + (5^2)^{\frac{1}{2}x+1} = 200$$

$$3(5^x)^2 + (5)^{x+2} = 200$$

$$3(5^x)^2 + (5)^x(5^2) = 200$$

$$3(5^x)^2 + 25(5)^x = 200$$

$$Let\ y = 5^x$$

$$3y^2 + 25y - 200 = 0$$

$$(3y + 40)(y - 5) = 0$$

$$y = -\frac{40}{3} \quad or \quad y = 5$$

$5^x = -\frac{40}{5}$ (ignored)

$$5^x = 5$$

$$x = 1$$

b) $x + 4 \leq x^2 + x < 12$

$x^2 + x \geq x + 4$

$x^2 + x - x - 4 \geq 0$

$x^2 - 4 \geq 0$

$(x + 2)(x - 2) \geq 0$

$x = -2 \quad or \quad x = 2$

And $x^2 + x < 12$

$x^2 + x - 12 < 0$

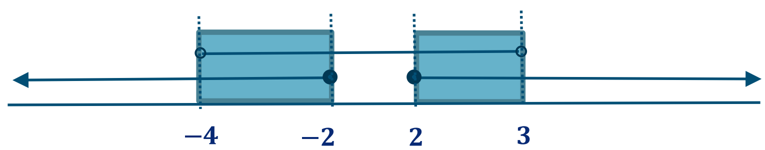
$(x + 4)(x - 3) < 0$

$x = -4 \quad or\ x = 3$

	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
$x+2$	-	+	+
$x-2$	-	-	+
	⊕	-	⊕

	$(-\infty,-4)$	$(-4,3)$	$(3,\infty)$
$x+4$	-	+	+
$x-3$	-	-	+
	+	⊝	+

$(-\infty,-2]\cup[2,\infty)$ And $(-4,3)$



$$(-4,-2]\cup[2,3)$$

Question A3

3. The sum of the first n terms of a sequence is given by $S_n = 2 + 3^{-4n}$.

   a. Find the value of constant c such that the n-th term is $c3^{-4n}$.

   b. Show that the sequence is a geometric series.

   c. Find the sum of the infinite series, S_∞.

SOLUTION

a) $S_n = 2 + 3^{-4n}$

$$T_n = S_n - S_{n-1}$$

$$c3^{-4n} = (2 + 3^{-4n}) - \left(2 + 3^{-4(n-1)}\right)$$

$$c3^{-4n} = 2 + 3^{-4n} - 2 - 3^{-4n+4}$$

$$c3^{-4n} = 3^{-4n} - 3^{-4n}3^4$$

$$c3^{-4n} = 3^{-4n} - 81(3^{-4n})$$

$$c3^{-4n} = 3^{-4n}(1 - 81)$$

$$c3^{-4n} = -80(3^{-4n})$$

$$c = -80$$

b) $T_n = -80\left(3^{-4n}\right)$

$$\frac{T_n}{T_{n-1}} = \frac{-80\left(3^{-4n}\right)}{-80\left[3^{-4(n-1)}\right]}$$

$$\frac{T_n}{T_{n-1}} = \frac{\left(3^{-4n}\right)}{\left[3^{-4n+4}\right]}$$

$$\frac{T_n}{T_{n-1}} = 3^{-4n+4n-4}$$

$$\frac{T_n}{T_{n-1}} = 3^{-4}$$

$$\frac{T_n}{T_{n-1}} = \frac{1}{81}$$

$$Since\ \frac{T_n}{T_{n-1}}\ is\ constant,\ therefore\ the\ sequence\ is\ geometry$$

c) $T_n = -80(3^{-4n})$

$$a = T_1 = -80(3^{-4(1)}) = -\frac{80}{81}$$

$$r = \frac{1}{81}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{\left(-\frac{80}{81}\right)}{1-\left(\frac{1}{81}\right)}$$

$$= \frac{\left(-\frac{80}{81}\right)}{\left(\frac{80}{81}\right)}$$

$$= -1$$

Question A4

4. Given matrix $A = \begin{bmatrix} 2 & 3 & 0 \\ -5 & 0 & 4 \\ 0 & 2 & 1 \end{bmatrix}$.

   a. Find the determinant of matrix A by expanding the first row.

   b. Calculate the adjoin of matrix A. Hence, find A^{-1}.

   c. Solve the equation $AX = B$, where $B = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, by using the answer obtained in part 4(b).

SOLUTION

a) $A = \begin{bmatrix} 2 & 3 & 0 \\ -5 & 0 & 4 \\ 0 & 2 & 1 \end{bmatrix}$

$$|A| = (2)\begin{vmatrix} 0 & 4 \\ 2 & 1 \end{vmatrix} - (3)\begin{vmatrix} -5 & 4 \\ 0 & 1 \end{vmatrix} + (0)\begin{vmatrix} -5 & 0 \\ 0 & 2 \end{vmatrix}$$

$$= (2)(0-8) - (3)(-5-0) + (0)|-10-0|$$

$$= -16 + 15 + 0$$

$$= -1$$

b) Adjoin of matrix A

$$Cofactor, C = \begin{pmatrix} +\begin{vmatrix} 0 & 4 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} -5 & 4 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} -5 & 0 \\ 0 & 2 \end{vmatrix} \\ -\begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} \\ +\begin{vmatrix} 3 & 0 \\ 0 & 4 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ -5 & 4 \end{vmatrix} & +\begin{vmatrix} 2 & 3 \\ -5 & 0 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} +(0-8) & -(-5-0) & +(-10-0) \\ -(3-0) & +(2-0) & -(4-0) \\ +(12-0) & -(8-0) & +(0+15) \end{pmatrix}$$

$$= \begin{pmatrix} -8 & 5 & -10 \\ -3 & 2 & -4 \\ 12 & -8 & 15 \end{pmatrix}$$

$$\boldsymbol{Adjoin\ A = C^T}$$

$$= \begin{pmatrix} -8 & -3 & 12 \\ 5 & 2 & -8 \\ -10 & -4 & 15 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} Adj\ A$$

$$= \frac{1}{-1}\begin{pmatrix} -8 & -3 & 12 \\ 5 & 2 & -8 \\ -10 & -4 & 15 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 3 & -12 \\ -5 & -2 & 8 \\ 10 & 4 & -15 \end{pmatrix}$$

c) $AX = B$

$$X = A^{-1}B$$

$$\begin{bmatrix} 2 & 3 & 0 \\ -5 & 0 & 4 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 8 & 3 & -12 \\ -5 & -2 & 8 \\ 10 & 4 & -15 \end{pmatrix}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= \begin{pmatrix} 8 + 6 - 24 \\ -5 - 4 + 16 \\ 10 + 8 - 30 \end{pmatrix}$$

$$= \begin{pmatrix} -10 \\ 7 \\ -12 \end{pmatrix}$$

$$\therefore x = -10, y = 7, z = -12$$

Question A5

5. Given $f(x) = \frac{1}{1+\frac{1}{1+\frac{1}{x}}}$.

   a. Simplify $f(x)$ and evaluate $f\left(\frac{1}{2}\right)$.

   b. The domain of $f(x)$ is a set of real number except three numbers. Determine the numbers.

SOLUTION

a) $f(x) = \frac{1}{1+\frac{1}{1+\frac{1}{x}}}$

$$= \frac{1}{1+\frac{1}{\frac{x+1}{x}}}$$

$$= \frac{1}{1+\frac{x}{x+1}}$$

$$= \frac{1}{\frac{x+1+x}{x+1}}$$

$$= \frac{x+1}{2x+1}$$

$$f\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)+1}{2\left(\frac{1}{2}\right)+1}$$

$$= \frac{\left(\frac{3}{2}\right)}{2}$$

$$= \frac{3}{4}$$

b) $f(x) = \frac{1}{1+\frac{1}{1+\frac{1}{x}}}$

$\frac{1}{x} \neq 0 \rightarrow x \neq 0$

$1 + \frac{1}{x} \neq 0 \rightarrow x \neq -1$

$1 + \frac{1}{1+\frac{1}{x}} \neq 0 \rightarrow 2x + 1 \neq 0 \rightarrow x \neq -\frac{1}{2}$

$\therefore x \neq 0; x \neq -1; x \neq -\frac{1}{2}$

Question B1

1. Solve the following:

   a. $log_2 2x = 2\, log_4(x+4)$

   b. $2\left|\frac{x-3}{2x-1}\right| \geq 1$

SOLUTION

a) $log_2 2x = 2\, log_4(x+4)$

$$log_2 2x = \frac{2log_2(x+4)}{log_2 4}$$

$$log_2 2x = \frac{2log_2(x+4)}{log_2 2^2}$$

$$log_2 2x = \frac{2log_2(x+4)}{2log_2 2}$$

$$log_2 2x = \frac{2log_2(x+4)}{2(1)}$$

$$log_2 2x = log_2(x+4)$$

$$2x = x+4$$

$$x = 4$$

b) $2\left|\frac{x-3}{2x-1}\right| \geq 1$

$$\left|\frac{x-3}{2x-1}\right| \geq \frac{1}{2}$$

$$\frac{x-3}{2x-1} \geq \frac{1}{2} \quad \text{OR} \quad \frac{x-3}{2x-1} \leq -\frac{1}{2}$$

$$\frac{x-3}{2x-1} - \frac{1}{2} \geq 0 \qquad \frac{x-3}{2x-1} + \frac{1}{2} \leq 0$$

$$\frac{2(x-3)-(2x-1)}{2(2x-1)} \geq 0 \qquad \frac{2(x-3)+(2x-1)}{2(2x-1)} \leq 0$$

$$\frac{2x-6-2x+1}{4x-2} \geq 0 \qquad \frac{2x-6+2x-1}{4x-2} \leq 0$$

$\frac{-5}{4x-2} \geq 0$

$4x - 2 < 0$

$x < \frac{2}{4}$

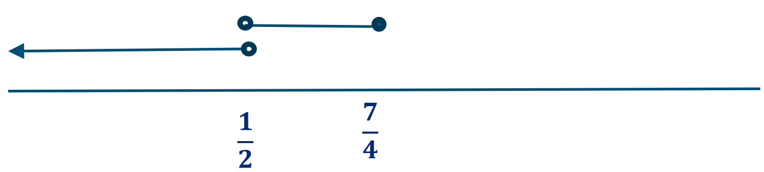
$x < \frac{1}{2}$

$\frac{4x-7}{4x-2} \leq 0$

$x = \frac{7}{4}; x = \frac{1}{2}$

	$\left(-\infty, \frac{1}{2}\right)$	$\left(\frac{1}{2}, \frac{7}{4}\right)$	$\left(\frac{7}{4}, \infty\right)$
$4x - 7$	-	-	+
$4x - 2$	-	+	+
	+	-	+

$\left(-\infty, \frac{1}{2}\right)$ OR $\left(\frac{1}{2}, \frac{7}{4}\right]$



$\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{7}{4}\right]$

Question B2

2. Given a function $f(x) = \ln(2x + 1)$

   a. State the domain and range of $f(x)$.

   b. Find the inverse function of $f(x)$ and state its domain and range. Hence, find the value of x for which $f^{-1}(x) = 0$.

   c. Sketch the graph of $f(x)$ and $f^{-1}(x)$ on the same coordinate axes.

SOLUTION

a) $f(x) = \ln(2x + 1)$

Domain:

$$D_f:\ 2x + 1 > 0$$

$$x > -\frac{1}{2}$$

$$D_f: \left(-\frac{1}{2}, \infty\right)$$

Range:

$$R_f = (-\infty, \infty)$$

b) $f(x) = \ln(2x + 1)$

$$f[f^{-1}(x)] = x$$

$$\ln[2f^{-1}(x) + 1] = x$$

$$2f^{-1}(x) + 1 = e^x$$

$$2f^{-1}(x) = e^x - 1$$

$$f^{-1}(x) = \frac{e^x - 1}{2}$$

Domain:

$$R_{f^{-1}} = R_f = (-\infty, \infty)$$

Range:

$$R_{f^{-1}} = D_f = \left(-\frac{1}{2}, \infty\right)$$

$$f^{-1}(x) = 0$$

$$\frac{e^x - 1}{2} = 0$$

$$e^x - 1 = 0$$

$$e^x = 1$$

$$x = ln\,1$$

$$x = 0$$

c)