SM015/2
MATEMATIK

2018/2019

Matriculation Programme Examination

a) Given $z_1 = 2 + 3i$ and $z_2 = 4 - 4i$. Express $\frac{(\overline{z_2})}{(\overline{z_1})} + \left[\left(\frac{i^3}{-z_2}\right)\right]$ in Cartesian form.

b) Solve

a. $\left(\frac{27}{125}\right)^2 \text{x} \left(\frac{25}{9}\right)^{4x} = \left(\frac{9}{25}\right)^{x-3} \text{x} \left(\frac{625}{81}\right)^2$

b. $\frac{1}{4-2x} \geq \frac{8}{x}$

c) a) The first three terms of a geometric series are $\left(3c - \frac{7}{2}\right), (3c - 2)$ and 6.

Determine the value of c. Hence, find the seventh term of this series.

b) Expand $\left(\frac{3}{2}x^2 - 1\right)^3$

d) a) Given the matrix $\begin{bmatrix} 1 & 3 & 4 \\ a+2b & 3 & 2 \\ 4 & a+b & 9 \end{bmatrix}$ such that $M_{11} = 7$ and $C_{12} = -1$,

calculate the values of a and b.

b) Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{bmatrix}, B = \begin{bmatrix} 7 & 13 & -6 \\ -1 & -7 & 2 \\ -2 & 2 & 0 \end{bmatrix}$ and $C = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

i. Find determinant of A by expanding first column.

ii. Evaluate $(A^2 - B^T)C$.

e) a) Given $f(x) = \left(\frac{5x+1}{4x}\right)$ and $g(x) = \frac{\sqrt{x+1}-2}{x^2-4}$. Find

i. the domain of g(x).

ii. $h(x), if\ (foh)(x) = x$

b) Given $p(x) = ln(3x+6)$ and $q(x) = \frac{e^x}{3} - 2$. Show that $p(x) and\ q(x)$ are inverses of each other.

f) The polynomial $P(x) = x^4 + ax^3 - 7x^2 - 4ax + b$ has a factor $(x+3)$ and remainder 60 when divided by $(x-3)$. Find the values of a and b. Hence, factorise $P(x)$ completely.

g) a) Express $12\cos\theta + 7\sin\theta$ in the form of $\text{R}\cos(\theta - \alpha)$, where $R > 0$ and $0° \leq \alpha \leq 90°$

b) Hence, show that the maximum value of $\frac{1}{12\cos\theta + 7\sin\theta + 15}$ is $\frac{1}{32}\left(15 + \sqrt{193}\right)$.

h) The function $g(x)$ is defined by

$$g(x) = \begin{cases} 2 & , x \leq 2 \\ \dfrac{x-2}{\sqrt{2x}-2} & , 2 < x \leq 8 \\ \dfrac{|8-x|}{x-8} & , x > 8 \end{cases}$$

Find

a) $\lim_{x \to 2^+} g(x)$

b) $\lim_{x \to 8^+} g(x)$

i) a) Find the derivative of $f(x) = \frac{6}{\sqrt{x}}$ using the first principle.

b) Find the value of $\frac{dy}{dx}$ when $x = 0$ for each of the following:

1. $y = \ln(9 - 2x)$
2. $y = \frac{e^{-3x}}{\sqrt{3x+1}}$

j) Given $f(x) = \frac{3x}{x^2+9}$, where $x > 0$. Find the coordinates of the stationary point and state it's nature.

1. Given $z_1 = 2 + 3i$ and $z_2 = 4 - 4i$. Express $\frac{(z_2)}{(\overline{z_1})} + \left[\left(\frac{i^3}{-z_2}\right)\right]$ in Cartesian form.

SOLUTION

$$z_1 = 2 + 3i$$

$$z_2 = 4 - 4i$$

$$\frac{(z_2)}{(\overline{z_1})} + \left[\left(\frac{i^3}{-z_2}\right)\right] = \frac{4 - 4i}{2 - 3i} + \left[\left(\frac{i^3}{-(4 - 4i)}\right)\right]$$

$$= \frac{4 - 4i}{2 - 3i} - \frac{i^3}{(4 - 4i)}$$

$$= \frac{(4 - 4i)(4 - 4i) - i^3(2 - 3i)}{(2 - 3i)(4 - 4i)}$$

$$= \frac{16 - 16i - 16i + 16i^2 - 2i^3 + 3i^4}{8 - 8i - 12i + 12i^2}$$

$$= \frac{16 - 16i - 16i + 16(-1) - 2(-i) + 3(1)}{8 - 20i + 12(-1)}$$

$$= \frac{16 - 16i - 16i - 16 + 2i + 3}{8 - 20i - 12}$$

$$= \frac{3 - 30i}{-4 - 20i}$$

$$= \frac{(3 - 30i)}{(-4 - 20i)} \cdot \frac{(-4 + 20i)}{(-4 + 20i)}$$

$$= \frac{-12 + 60i + 120i - 600i^2}{16 - 80i + 80i - 400i^2}$$

$$= \frac{-12 + 60i + 120i - 600(-1)}{16 - 80i + 80i - 400(-1)}$$

$$= \frac{588 + 180i}{416}$$

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$= \frac{588}{416} + \frac{180}{416}i$$

$$= \frac{147}{104} + \frac{45}{104}i$$

2. Solve

a. $\left(\frac{27}{125}\right)^2 \text{ x } \left(\frac{25}{9}\right)^{4x} = \left(\frac{9}{25}\right)^{x-3} \text{ x } \left(\frac{625}{81}\right)^2$

b. $\frac{1}{4-2x} \geq \frac{8}{x}$

SOLUTION

a. $\left(\frac{27}{125}\right)^2 \text{ x } \left(\frac{25}{9}\right)^{4x} = \left(\frac{9}{25}\right)^{x-3} \text{ x } \left(\frac{625}{81}\right)^2$

$$\frac{\left(\frac{25}{9}\right)^{4x}}{\left(\frac{9}{25}\right)^{x-3}} = \frac{\left(\frac{625}{81}\right)^2}{\left(\frac{27}{125}\right)^2}$$

$$\frac{\left(\frac{25}{9}\right)^{4x}}{\left(\frac{25}{9}\right)^{3-x}} = \frac{\left[\left(\frac{5}{3}\right)^4\right]^2}{\left[\left(\frac{3}{5}\right)^3\right]^2}$$

$$\frac{\left[\left(\frac{5}{3}\right)^2\right]^{4x}}{\left[\left(\frac{5}{3}\right)^2\right]^{3-x}} = \frac{\left(\frac{5}{3}\right)^8}{\left(\frac{3}{5}\right)^6}$$

$$\frac{\left(\frac{5}{3}\right)^{8x}}{\left(\frac{5}{3}\right)^{6-2x}} = \frac{\left(\frac{5}{3}\right)^8}{\left(\frac{5}{3}\right)^{-6}}$$

$$\left(\frac{5}{3}\right)^{8x-(6-2x)} = \left(\frac{5}{3}\right)^{8-(-6)}$$

$$\left(\frac{5}{3}\right)^{10x-6} = \left(\frac{5}{3}\right)^{14}$$

$$\left(\frac{a}{b}\right)^c = \left(\frac{b}{a}\right)^{-c}$$

$$\left(\frac{9}{25}\right)^{x-3} = \left(\frac{25}{9}\right)^{3-x}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$10x - 6 = 14$

$x = 2$

b. $\frac{1}{4-2x} \geq \frac{8}{x}$

$$\frac{1}{4-2x} - \frac{8}{x} \geq 0$$

$$\frac{x - 8(4-2x)}{x(4-2x)} \geq 0$$

$$\frac{x - 32 + 16x}{x(4-2x)} \geq 0$$

$$\frac{17x - 32}{x(4-2x)} \geq 0$$

Critical value:

$x = \frac{32}{17}$ $\quad$ $x = 0$ $\quad$ $x = 2$

x	$(-\infty, 0)$	$\left(0, \frac{32}{17}\right)$	$\left(\frac{32}{17}, 2\right)$	$(2, \infty)$
$17x - 32$	-	-	+	+
$(4 - 2x)$	+	+	+	-
x	-	+	+	+
$\frac{17x-32}{x(4-2x)}$	⊕	-	⊕	-

Solution: $\left\{x : x < 0 \cup \frac{32}{17} \leq x < 2\right\}$

3. a) The first three terms of a geometric series are $\left(3c-\frac{7}{2}\right),(3c-2)\ and\ 6$.

Determine the value of c. Hence, find the seventh term of this series.

b) Expand $\left(\frac{3}{2}x^2-1\right)^3$

SOLUTION

a) Geometric series

$$\left(3c-\frac{7}{2}\right),(3c-2)\ and\ 6$$

$$\frac{T_2}{T_1}=\frac{T_3}{T_2}$$

$$\frac{3c-2}{3c-\frac{7}{2}}=\frac{6}{3c-2}$$

$$(3c-2)^2=6\left(3c-\frac{7}{2}\right)$$

$$9c^2-12c+4=18c-21$$

$$9c^2-30c+25=0$$

$$(3c-5)(3c-5)=0$$

$$c=\frac{5}{3}$$

$$a=3c-\frac{7}{2}$$

$$=3\left(\frac{5}{3}\right)-\frac{7}{2}$$

$$= \frac{3}{2}$$

$$r = \frac{6}{3c-2}$$

$$= \frac{6}{3\left(\frac{5}{3}\right)-2}$$

$$= 2$$

$$T_7 = ar^6$$

$$= \left(\frac{3}{2}\right)(2)^6$$

$$= 96$$

b) $$\left(\frac{3}{2}x^2-1\right)^3 = \binom{3}{0}\left(\frac{3x^2}{2}\right)^3(-1)^0 + \binom{3}{1}\left(\frac{3x^2}{2}\right)^2(-1)^1 + \binom{3}{2}\left(\frac{3x^2}{2}\right)^1(-1)^2 + \binom{3}{3}\left(\frac{3x^2}{2}\right)^0(-1)^3$$

$$= (1)\left(\frac{27x^6}{8}\right)(1) + (3)\left(\frac{9x^4}{4}\right)(-1) + (3)\left(\frac{3x^2}{2}\right)(1) + (1)(1)(-1)$$

$$= \frac{27}{8}x^6 - \frac{27}{4}x^4 + \frac{9}{2}x^2 - 1$$

i. a) Given the matrix $\begin{bmatrix} 1 & 3 & 4 \\ a+2b & 3 & 2 \\ 4 & a+b & 9 \end{bmatrix}$ such that $M_{11} = 7$ and $C_{12} = -1$,

calculate the values of a and b.

b) Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{bmatrix}, B = \begin{bmatrix} 7 & 13 & -6 \\ -1 & -7 & 2 \\ -2 & 2 & 0 \end{bmatrix}$ $and\ C = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

i. Find determinant of A by expanding first column.

ii. Evaluate $(A^2 - B^T)C$.

SOLUTION

a) $\begin{bmatrix} 1 & 3 & 4 \\ a+2b & 3 & 2 \\ 4 & a+b & 9 \end{bmatrix}$

$$M_{11} = \begin{vmatrix} 3 & 2 \\ a+b & 9 \end{vmatrix}$$

$$= 27 - 2a - 2b$$

$$27 - 2a - 2b = 7$$

$$2a + 2b = 20$$

$a + b = 10$ .... (1)

$$C_{12} = (-1)^{1+2} \begin{vmatrix} a+2b & 2 \\ 4 & 9 \end{vmatrix}$$

$$= (-1)[9a + 18b - 8]$$

$= -9a - 18b + 8$

$-9a - 18b + 8 = -1$

$9a + 18b = 9$

$a + 2b = 1$ .... (2)

$(2) - (1)$

$b = 1 - 10 = -9$

$a + (-9) = 10$

$a = 19$

$\therefore a = 19, b = -9$

bi)

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{bmatrix}, B = \begin{bmatrix} 7 & 13 & -6 \\ -1 & -7 & 2 \\ -2 & 2 & 0 \end{bmatrix} \text{ and } C = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$|A| = (1)\begin{vmatrix} 3 & 2 \\ 10 & 9 \end{vmatrix} - (1)\begin{vmatrix} 3 & 4 \\ 10 & 9 \end{vmatrix} + (4)\begin{vmatrix} 3 & 4 \\ 3 & 2 \end{vmatrix}$$

$= (1)[27 - 20] - (1)[27 - 40] + (4)[6 - 12]$

$= 7 + 13 - 24$

$= -4$

bii) $(A^2 - B^T)C = \left[\begin{pmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{pmatrix}\begin{pmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{pmatrix} - \begin{pmatrix} 7 & -1 & -2 \\ 13 & -7 & 2 \\ -6 & 2 & 0 \end{pmatrix}\right]\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$$= \left[\begin{pmatrix} 20 & 52 & 46 \\ 12 & 32 & 28 \\ 50 & 132 & 117 \end{pmatrix} - \begin{pmatrix} 7 & -1 & -2 \\ 13 & -7 & 2 \\ -6 & 2 & 0 \end{pmatrix}\right]\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$= \left[\begin{pmatrix} 13 & 53 & 48 \\ -1 & 39 & 26 \\ 56 & 130 & 117 \end{pmatrix}\right]\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 162 \\ 90 \\ 420 \end{pmatrix}$$

5. a) Given $f(x) = \left(\frac{5x+1}{4x}\right)$ and $g(x) = \frac{\sqrt{x+1}-2}{x^2-4}$. Find

   i. The domain of g(x).

   ii. $h(x), if\ (foh)(x) = x$

   b) Given $p(x) = ln(3x+6)$ and $q(x) = \frac{e^x}{3} - 2$. Show that $p(x) and\ q(x)$ are inverses of each other.

SOLUTION

$$f(x) = \left(\frac{5x+1}{4x}\right)$$

$$g(x) = \frac{\sqrt{x+1}-2}{x^2-4}$$

5ai) $Domain\ of\ g(x)$

$x + 1 \geq 0$ and $x^2 - 4 \neq 0$

$x \geq -1$ and $x \neq \pm 2$

$\therefore D_f: [-1,2) \cup (2, \infty)$

$5aii)$ $(foh)(x) = x$

$$f[h(x)] = x$$

$$\left[\frac{5h(x)+1}{4h(x)}\right] = x$$

$$5h(x) + 1 = 4xh(x)$$

$$5h(x) - 4xh(x) = -1$$

$$h(x)[5 - 4x] = -1$$

$$h(x) = \frac{-1}{5-4x}$$

5b) Given $p(x) = ln(3x+6)$ and $q(x) = \frac{e^x}{3} - 2$

$$p[q(x)] = ln\left[3\left(\frac{e^x}{3} - 2\right) + 6\right]$$

$$= ln[e^x - 6 + 6]$$

$$= ln[e^x]$$

$$= xln[e]$$

$$= x$$

$$q[p(x)] = \frac{e^{ln(3x+6)}}{3} - 2$$

$$= \frac{3x+6}{3} - 2$$

$$= \frac{3x+6-6}{3}$$

$$= x$$

$Since\ p[q(x)] = q[p(x)] = x, therefore\ p(x) and\ q(x) are\ inverses\ of\ each\ other$

6. The polynomial $P(x) = x^4 + ax^3 - 7x^2 - 4ax + b$ has a factor $(x + 3)$ and remainder 60 when divided by $(x - 3)$. Find the values of a and b. Hence, factorise $P(x)$ completely.

SOLUTION

$P(x) = x^4 + ax^3 - 7x^2 - 4ax + b$

$P(-3) = 0$

$P(3) = 60$

$P(-3) = (-3)^4 + a(-3)^3 - 7(-3)^2 - 4a(-3) + b = 0$

$81 - 27a - 63 + 12a + b = 0$

$15a - b = 18$ .... (1)

$P(3) = (3)^4 + a(3)^3 - 7(3)^2 - 4a(3) + b = 60$

$81 + 27a - 63 - 12a + b = 60$

$15a + b = 42$ .... (2)

$(2) - (1)$

$2b = 24$

$b = 12$

$15a - 12 = 18$

$a = 2$

$\therefore a = 2, b = 12$

$P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$

$$
\begin{array}{r|l}
 & x^3 - x^2 - 4x + 4 \\
\hline
x+3 & x^4 + 2x^3 - 7x^2 - 8x + 12 \\
 & x^4 + 3x^3 \\
\hline
 & -x^3 - 7x^2 - 8x + 12 \\
 & -x^3 - 3x^2 \\
\hline
 & -4x^2 - 8x + 12 \\
 & -4x^2 - 12x \\
\hline
 & 4x + 12 \\
 & 4x + 12 \\
\hline
 & 0
\end{array}
$$

$P(x) = (x+3)(x^3 - x^2 - 4x + 4)$

$= (x+3)[x^2(x-1) - 4(x-1)]$

$= (x+3)(x-1)[x^2 - 4]$

$= (x+3)(x-1)(x+2)(x-2)$

7. a) Express $12\cos\theta + 7\sin\theta$ in the form of $\mathrm{R}\cos(\theta - \alpha)$, where $R > 0$ and $0° \leq \alpha \leq 90°$

b) Hence, show that the maximum value of $\frac{1}{12\cos\theta+7\sin\theta+15}$ is $\frac{1}{32}(15 + \sqrt{193})$.

SOLUTION

7a) $12\cos\theta + 7\sin\theta = \mathrm{R}\cos(\theta - \alpha)$

$12\cos\theta + 7\sin\theta = R[\cos\theta\cos\alpha + \sin\theta\sin\alpha]$

$12\cos\theta + 7\sin\theta = \mathrm{R}\cos\theta\cos\alpha + \mathrm{R}\sin\theta\sin\alpha$

$R\cos\alpha = 12$ .... (1)

$R\sin\alpha = 7$ .... (2)

$(1)^2 + (2)^2$

$R^2 cos^2\alpha + R^2 sin^2\alpha = 12^2 + 7^2$

$R^2(cos^2\alpha + sin^2\alpha) = 193\mathrm{c}$

$R^2(1) = 193$

$R = \sqrt{193}$

$(2) \div (1)$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{7}{12}$

$\tan\alpha = \frac{7}{12}$

$\alpha = 30.3°$

$12\cos\theta + 7\sin\theta = \sqrt{193}\cos(\theta - 30.3°)$

7b) $\frac{1}{12\cos\theta+7\sin\theta+15} = \frac{1}{\left(\sqrt{193}\cos(\theta-30.3°)\right)+15}$

$$-1 \le \cos(\theta - 30.3°) \le 1$$

$$-\sqrt{193} \le \sqrt{193}\cos(\theta - 30.3°) \le \sqrt{193}$$

$$-\sqrt{193} + 15 \le \sqrt{193}\cos(\theta - 30.3°) + 15 \le \sqrt{193} + 15$$

$$\frac{1}{\sqrt{193}+15} \le \frac{1}{\sqrt{193}\cos(\theta-30.3°)+15} \le \frac{1}{-\sqrt{193}+15}$$

$$\frac{1}{\sqrt{193}+15} \le \frac{1}{12\cos\theta+7\sin\theta+15} \le \frac{1}{-\sqrt{193}+15}$$

The Maximum value of $\frac{1}{12\cos\theta+7\sin\theta+15}$

$$\frac{1}{-\sqrt{193}+15} = \frac{1}{\left(15-\sqrt{193}\right)}\cdot\frac{\left(15+\sqrt{193}\right)}{\left(15+\sqrt{193}\right)}$$

$$= \frac{15+\sqrt{193}}{225+15\sqrt{193}-15\sqrt{193}-193}$$

$$= \frac{15+\sqrt{193}}{32}$$

$$= \frac{1}{32}\left(15+\sqrt{193}\right)$$

8. The function $g(x)$ is defined by

$$g(x)=\begin{cases} 2 & ,x\le 2 \\ \dfrac{x-2}{\sqrt{2x}-2} & ,2<x\le 8 \\ \dfrac{|8-x|}{x-8} & ,x>8 \end{cases}$$

Find

a) $\lim\limits_{x\to 2^+} g(x)$

b) $\lim\limits_{x\to 8^+} g(x)$

SOLUTION

a) $\lim\limits_{x\to 2^+} g(x)=\lim\limits_{x\to 2^+}\left(\frac{x-2}{\sqrt{2x}-2}\right)$

$$=\lim_{x\to 2^+}\left(\frac{x-2}{\sqrt{2x}-2}\right)\left(\frac{\sqrt{2x}+2}{\sqrt{2x}+2}\right)$$

$$=\lim_{x\to 2^+}\left(\frac{(x-2)(\sqrt{2x}+2)}{2x+2\sqrt{2x}-2\sqrt{2x}-4}\right)$$

$$=\lim_{x\to 2^+}\left(\frac{(x-2)(\sqrt{2x}+2)}{2x-4}\right)$$

$$=\lim_{x\to 2^+}\left(\frac{(x-2)(\sqrt{2x}+2)}{2(x-2)}\right)$$

$$=\lim_{x\to 2^+}\left[\frac{(\sqrt{2x}+2)}{2}\right]$$

$$=\frac{\sqrt{2(2)}+2}{2}$$

$$=2$$

b) $|8-x| = \begin{cases} 8-x & 8-x \geq 0 \\ -(8-x) & 8-x < 0 \end{cases}$

$$= \begin{cases} 8-x & x \leq 8 \\ -(8-x) & x > 8 \end{cases}$$

$$\lim_{x \to 8^+} g(x) = \lim_{x \to 8^+} \frac{|8-x|}{x-8}$$

$$= \lim_{x \to 8^+} \frac{-(8-x)}{x-8}$$

$$= \lim_{x \to 8^+} \frac{x-8}{x-8}$$

$$= \lim_{x \to 8^+} 1$$

$$= 1$$

9. a) Find the derivative of $f(x) = \frac{6}{\sqrt{x}}$ using the first principle.

b) Find the value of $\frac{dy}{dx}$ when $x = 0$ for each of the following:

i. $y = \ln(9 - 2x)$

ii. $y = \frac{e^{-3x}}{\sqrt{3x+1}}$

SOLUTION

a) $f(x) = \frac{6}{\sqrt{x}}$

$$f(x+h) = \frac{6}{\sqrt{x+h}}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{6}{\sqrt{x+h}} - \frac{6}{\sqrt{x}}}{h}$$

$$= \lim_{h\to 0} \left(\frac{6}{\sqrt{x+h}} - \frac{6}{\sqrt{x}}\right).\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{6\sqrt{x} - 6\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right).\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \left[\frac{6\left(\sqrt{x} - \sqrt{x+h}\right)}{\sqrt{x}\sqrt{x+h}}\right].\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \left[\frac{6\left(\sqrt{x} - \sqrt{x+h}\right)\left(\sqrt{x} + \sqrt{x+h}\right)}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)}\right].\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \left[\frac{6\left(x + \sqrt{x}\sqrt{x+h} - \sqrt{x}\sqrt{x+h} - (x+h)\right)}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)}\right].\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}\left[\frac{6(-h)}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}\right]\cdot\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}\left[\frac{-6}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}\right]$$

$$= \frac{-6}{\sqrt{x}\sqrt{x+0}\left(\sqrt{x}+\sqrt{x+0}\right)}$$

$$= \frac{-6}{\sqrt{x}\sqrt{x}\left(\sqrt{x}+\sqrt{x}\right)}$$

$$= \frac{-6}{x\left(2\sqrt{x}\right)}$$

$$= \frac{-3}{x^{\frac{3}{2}}}$$

bi) $y = \ln(9-2x)$

$$\frac{dy}{dx} = \frac{1}{9-2x}\frac{d}{dx}(9-2x)$$

$$\frac{dy}{dx} = \frac{1}{9-2x}(-2)$$

$$\frac{dy}{dx} = \frac{-2}{9-2x}$$

$When\ x = 0$

$$\frac{dy}{dx} = \frac{-2}{9-0}$$

$$= -\frac{2}{9}$$

bii) $y = \frac{e^{-3x}}{\sqrt{3x+1}}$

$u = e^{-3x}$

$u' = -3e^{-3x}$

$v = \sqrt{3x+1} = (3x+1)^{\frac{1}{2}}$

$$v' = \frac{1}{2}(3x+1)^{-\frac{1}{2}}\frac{d}{dx}(3x+1)$$

$$= \frac{3}{2(3x+1)^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{(3x+1)^{\frac{1}{2}}(-3e^{-3x}) - e^{-3x}\left(\frac{3}{2(3x+1)^{\frac{1}{2}}}\right)}{\left[(3x+1)^{\frac{1}{2}}\right]^2}$$

$When\ x = 0;$

$$\frac{dy}{dx} = \frac{(0+1)^{\frac{1}{2}}(-3e^{0}) - e^{0}\left(\frac{3}{2(0+1)^{\frac{1}{2}}}\right)}{\left[(0+1)^{\frac{1}{2}}\right]^2}$$

$$= \frac{-3-\frac{3}{2}}{1}$$

$$= -\frac{9}{2}$$

10. Given $f(x) = \frac{3x}{x^2+9}$, where $x > 0$. Find the coordinates of the stationary point and state it's nature.

SOLUTION

$$f(x) = \frac{3x}{x^2 + 9}$$

$u = 3x \qquad v = x^2 + 9$

$u' = 3 \qquad v' = 2x$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x^2 + 9)(3) - (3x)(2x)}{(x^2 + 9)^2}$$

$$= \frac{3x^2 + 27 - 6x^2}{(x^2 + 9)^2}$$

$$= \frac{-3x^2 + 27}{(x^2 + 9)^2}$$

$u = -3x^2 + 27 \qquad v = (x^2 + 9)^2$

$u' = -9x$

$$v' = 2(x^2 + 9)\frac{d}{dx}(x^2 + 9)$$

$$= 2(x^2 + 9)(2x)$$

$$= 4x(x^2 + 9)$$

$$f''(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x^2+9)^2(-9x) - (-3x^2+27)4x(x^2+9)}{(x^2+9)^4}$$

$$= \frac{-9x(x^2+9)^2 - 4x(-3x^2+27)(x^2+9)}{(x^2+9)^4}$$

$Let\ f'(x) = 0$

$$\frac{-3x^2+27}{(x^2+9)^2} = 0$$

$$-3x^2 + 27 = 0$$

$$3x^2 = 27$$

$$x^2 = 9$$

$$x = \pm 3$$

$Since\ x > 0\ , x = 3$

$\boldsymbol{When\ x = 3}$

$$f(x) = \frac{3x}{x^2+9}$$

$$= \frac{9}{9+9}$$

$$= \frac{1}{2}$$

$\left(3, \frac{1}{2}\right)$ *is a stationary point*

$$f''(x) = \frac{-9x(x^2+9)^2 - 4x(-3x^2+27)(x^2+9)}{(x^2+9)^4}$$

$$= \frac{-27(9+9)^2 - 12(-27+27)(9+9)}{(9+9)^4}$$

$$= \frac{-27(9+9)^2 - 0}{(9+9)^4}$$

$$< 0\ (Max)$$

$$\therefore \left(3, \frac{1}{2}\right) \text{ is a maximum point}$$