KEMENTERIAN
PENDIDIKAN
MALAYSIA

QS 015/1

Matriculation Programme Examination

Semester I

Session 2016/2017

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1. Solve for p and q where $p \neq q$, such that $\frac{(p+qi)}{3i} = (3 + \sqrt{-16} - i^3)$.

SOLUTION

$$\frac{(p+qi)}{3i} = (3 + \sqrt{-16} - i^3)$$

$$p + qi = (3 + \sqrt{-16} - i^3)(3i)$$

$$p + qi = (3 + \sqrt{16}\sqrt{-1} - i^3)(3i)$$

$$p + qi = (3 + 4i - i^3)(3i)$$

$$p + qi = 9i + 12i^2 - 3i^4$$

$$p + qi = 9i + 12(-1) - 3(1)$$

$$p + qi = 9i - 15$$

$$p + qi = -15 + 9i$$

$$\boldsymbol{p = -15, \quad q = 9}$$

2. Determine the values of x which satisfy the equation $3^{2x-1} = 4(3^x) - 9$.

SOLUTION

$$3^{2x-1} = 4(3^x) - 9$$

$$3^{2x}3^{-1} = 4(3^x) - 9$$

$$(3^x)^2\left(\frac{1}{3}\right) = 4(3^x) - 9$$

$$\left(\frac{1}{3}\right)(3^x)^2 = 4(3^x) - 9$$

$$(3^x)^2 = 12(3^x) - 27$$

$Let\ y = 3^x$

$$(y)^2 = 12(y) - 27$$

$$y^2 - 12y + 27 = 0$$

$$(y-3)(y-9) = 0$$

$y = 3$ $y = 9$

$3^x = 3$ $3^x = 9$

$x = 1$ $x = 2$

3. The seventh term of a geometric series is 16, the fifth term is 8 and the sum of the first ten terms is positive. Find the first term and the common ratio. Hence, show that $S_{12} = 126(\sqrt{2} + 1)$.

SOLUTION

Geometric series

$\boldsymbol{Geometric\ series \Rightarrow T_n = ar^{n-1}}$

$T_7 = ar^6 = 16$ …… (1)

$T_5 = ar^4 = 8$ …… (2)

$(1) \div (2)$

$$\frac{ar^6}{ar^4} = \frac{16}{8}$$

$$r^2 = 2$$

$$r = \pm\sqrt{2}$$

$When\, r = \pm\sqrt{2}$

$$a(\pm\sqrt{2})^4 = 8$$

$$4a = 8$$

$$a = 2$$

$$\textit{Geometric series} \rightarrow \boldsymbol{S_n = \frac{a(r^n - 1)}{r - 1}}$$

Given that $S_n > 0$

Given that $S_n > 0$

$$\boldsymbol{S_{10} = \frac{a\left(r^{10} - 1\right)}{r - 1}}$$

$When\, a = 2;\;\; r = \sqrt{2}$

$$S_{10} = \frac{2\left[\left(\sqrt{2}\right)^{10} - 1\right]}{\sqrt{2} - 1}$$

$$= \frac{2[32 - 1]}{\sqrt{2} - 1}$$

$$= \frac{62}{\sqrt{2} - 1}$$

$$= 149.68$$

$When\, a = 2;\;\; r = -\sqrt{2}$

$$S_{10} = \frac{2\left[\left(-\sqrt{2}\right)^{10} - 1\right]}{-\sqrt{2} - 1}$$

$$= \frac{2[32 - 1]}{-\sqrt{2} - 1}$$

$$= \frac{62}{-\sqrt{2} - 1}$$

$$= -25.68$$

$Given\ that\ S_n > 0, \rightarrow\;\; a = 2 \qquad r = \sqrt{2}$

$$S_{12} = \frac{2\left[\left(\sqrt{2}\right)^{12} - 1\right]}{\sqrt{2} - 1}$$

$$= \frac{2[64 - 1]}{\sqrt{2} - 1}$$

$$= \frac{126}{\sqrt{2} - 1}$$

$$= \frac{126}{\sqrt{2} - 1}\ x\ \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$= \frac{126\left(\sqrt{2} + 1\right)}{2 + \sqrt{2} - \sqrt{2} - 1}$$

$$= \frac{126\left(\sqrt{2} + 1\right)}{1}$$

$$= 126\left(\sqrt{2} + 1\right)$$

4. If A and B are $2\ x\ 2$ matrices where $B \neq I_2$, such that $(A+B)^2 = A^2 + 2AB + B^2$, deduce that $B = A^{-1}$. If $A = \begin{bmatrix} 1 & 2 \\ 9 & -1 \end{bmatrix}$, find B.

SOLUTION

$$(A+B)^2 = A^2 + 2AB + B^2$$

$$(A+B)(A+B) = A^2 + 2AB + B^2$$

$$A^2 + AB + BA + B^2 = A^2 + 2AB + B^2$$

$$AB + BA = 2AB$$

$$BA = 2AB - AB$$

$$BA = AB$$

$$\therefore B = A^{-1}$$

$If\ AB = BA\ \ then\ B^{-1} = A$

$$A = \begin{bmatrix} 1 & 2 \\ 9 & -1 \end{bmatrix}$$

$$B = A^{-1} = \frac{1}{(1)(-1)-(2)(9)}\begin{bmatrix} -1 & -2 \\ -9 & 1 \end{bmatrix}$$

$$= \frac{1}{-19}\begin{bmatrix} -1 & -2 \\ -9 & 1 \end{bmatrix}$$

$$= \frac{1}{-19}\begin{bmatrix} -1 & -2 \\ -9 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{9}{19} & \frac{1}{19} \end{bmatrix}$$

5. (a) Obtain the expansion for $\left(1-\frac{x}{4}\right)^{\frac{1}{4}}$ in ascending powers of x up to the term x^3. State the interval for x such that the expansion $\left(1-\frac{x}{4}\right)^{\frac{1}{4}}$ is valid. Hence, obtain the simplest form of the expansion $(16-4x)^{\frac{1}{4}}$.

(b) Write $\sqrt[4]{12}$ in the form of $K\left[\left(1-\frac{x}{4}\right)^{\frac{1}{4}}\right]$. Hence, approximate $\sqrt[4]{12}$ correct to three decimal places.

SOLUTION

a) $\left(1-\frac{x}{4}\right)^{\frac{1}{4}} = 1+\frac{\left(\frac{1}{4}\right)}{1!}\left(-\frac{x}{4}\right)^1+\frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)}{2!}\left(-\frac{x}{4}\right)^2+\frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-2\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

$$=1+\left(\frac{1}{4}\right)\left(-\frac{x}{4}\right)+\frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)}{2x1}\left(\frac{x^2}{16}\right)+\frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)}{3x2x1}\left(-\frac{x^3}{64}\right)+\cdots$$

$$=1-\frac{x}{16}-\frac{3}{32}\left(\frac{x^2}{16}\right)-\frac{21}{384}\left(\frac{x^3}{64}\right)+\cdots$$

$$=1-\frac{1}{16}x-\frac{3}{512}x^2-\frac{7}{8192}x^3+\cdots$$

The interval for x such that the expansion $\left(1-\frac{x}{4}\right)^{\frac{1}{4}}$ is valid

$$\left|\frac{x}{4}\right|<1$$

$$-1<\frac{x}{4}<1$$

$$-4<x<4$$

The simplest form of the expansion $(\mathbf{16-4x})^{\frac{1}{4}}$

$$(16-4x)^{\frac{1}{4}} = \left[16\left(1-\frac{4x}{16}\right)\right]^{\frac{1}{4}}$$

$$= 16^{\frac{1}{4}}\left(1-\frac{x}{4}\right)^{\frac{1}{4}}$$

$$= 2\left(1-\frac{x}{4}\right)^{\frac{1}{4}}$$

$$= 2\left[1-\frac{1}{16}x-\frac{3}{512}x^2-\frac{7}{8192}x^3+\cdots\right]$$

$$= 2-\frac{1}{8}x-\frac{3}{256}x^2-\frac{7}{4096}x^3+\cdots$$

b) $\sqrt[4]{\mathbf{12}}$ ***in the form of*** $K\left[\left(\mathbf{1}-\frac{x}{4}\right)^{\frac{1}{4}}\right]$

$$\sqrt[4]{12} = 12^{\frac{1}{4}}$$

$$= (16-4)^{\frac{1}{4}}$$

$$= \left[16\left(1-\frac{4}{16}\right)\right]^{\frac{1}{4}}$$

$$= 16^{\frac{1}{4}}\left(1-\frac{1}{4}\right)^{\frac{1}{4}}$$

$$= 2\left(1-\frac{1}{4}\right)^{\frac{1}{4}} \xrightarrow{\text{Compare}} \boxed{2\left(1-\frac{x}{4}\right)^{\frac{1}{4}} = 2-\frac{1}{8}x-\frac{3}{256}x^2-\frac{7}{4096}x^3}$$

$$= 2-\frac{1}{8}(1)-\frac{3}{256}(1)^2-\frac{7}{4096}(1)^3+\cdots$$

$$\approx 1.862$$

6. Given $f(x) = \frac{x^2+1}{5}$, $x \geq 0$.

(a) Determine $f^{-1}(x)$. Hence, if $f\big(g(x)\big) = \frac{1}{5}\big(e^{2(3x-1)} + 1\big)$, show that $g(x) = e^{3x-1}$

(b) Evaluate $g\big(f(2)\big)$ correct to three decimal places.

(c) Assume that the domain for $g(x)$ is $x \geq 0$, determine $g^{-1}(x)$ and state its domain and range.

SOLUTION

a) $f(x) = \frac{x^2+1}{5}$, $x \geq 0$

Method I	**Method II**
$f(x) = \dfrac{x^2+1}{5}$ $f[f^{-1}(x)] = x$ $\dfrac{[f^{-1}(x)]^2+1}{5} = x$ $[f^{-1}(x)]^2 + 1 = 5x$ $[f^{-1}(x)]^2 = 5x - 1$ $f^{-1}(x) = \sqrt{5x-1}$	$y = \dfrac{x^2+1}{5}$ $5y = x^2 + 1$ $x^2 = 5y - 1$ $x = \sqrt{5y-1}$ $f^{-1}(x) = \sqrt{5x-1}$

$$f\big(g(x)\big) = \frac{1}{5}\big(e^{2(3x-1)} + 1\big)$$

$$\frac{[g(x)]^2 + 1}{5} = \frac{1}{5}\big(e^{2(3x-1)} + 1\big)$$

$$[g(x)]^2 + 1 = e^{2(3x-1)} + 1$$

$$[g(x)]^2 = e^{2(3x-1)}$$

$$g(x) = \left[e^{2(3x-1)}\right]^{\frac{1}{2}}$$

$$g(x) = e^{3x-1}$$

b) Evaluate $g(f(2))$

$$f(2) = \frac{2^2 + 1}{5}$$

$$= 1$$

$$g(f(2)) = g(1)$$

$$= e^{3(1)-1}$$

$$= e^2$$

c) $g(x) = e^{3x-1}$

Method I	**Method II**
$g(x) = e^{3x-1}$ $g[g^{-1}(x)] = x$ $e^{3[g^{-1}(x)]-1} = x$ $\ln e^{3[g^{-1}(x)]-1} = \ln x$ $3[g^{-1}(x)] - 1 = \ln x$ $3[g^{-1}(x)] = \ln x + 1$ $g^{-1}(x) = \frac{\ln x + 1}{3}$	$g(x) = e^{3x-1}$ $y = e^{3x-1}$ $\ln y = \ln e^{3x-1}$ $\ln y = 3x - 1$ $3x = \ln y + 1$ $x = \frac{\ln y + 1}{3}$ $g^{-1}(x) = \frac{\ln x + 1}{3}$

$g(x) = e^{3x-1}; x \geq 0$

$D_{g(x)}: \; x \geq 0$

$R_{g(x)}: \; y \geq e^{3(0)-1}$

$y \geq e^{-1}$

$y \geq \frac{1}{e}$

$D_{g^{-1}(x)} = R_{g(x)}$

$R_{g^{-1}(x)} = D_{g(x)}$

$D_{g^{-1}(x)}: \; x \geq \frac{1}{e}$

$R_{g^{-1}(x)}: \; y \geq 0$

7. (a) If $\sqrt{7-3\sqrt{5}} = \sqrt{x} - \sqrt{y}$, determine the values of x and y.

(b) Solve the equation $log_2 x - log_4(3x+4) = 0$.

SOLUTION

a) $\sqrt{7-3\sqrt{5}} = \sqrt{x} - \sqrt{y}$

$$\left[\sqrt{7-3\sqrt{5}}\right]^2 = \left[\sqrt{x} - \sqrt{y}\right]^2$$

$$7-3\sqrt{5} = \sqrt{x}^2 + \sqrt{y}^2 - 2\sqrt{x}\sqrt{y}$$

$$7-3\sqrt{5} = x + y - 2\sqrt{xy}$$

$$x + y = 7$$

$y = 7 - x$ .... (1)

$$2\sqrt{xy} = 3\sqrt{5}$$

$$\sqrt{4xy} = \sqrt{9x5}$$

$4xy = 45$ .... (2)

Substitute (1) into (2)

$$4x(7-x) = 45$$

$$28x - 4x^2 = 45$$

$$4x^2 - 28x + 45 = 0$$

$$(2x-9)(2x-5) = 0$$

$x = \frac{9}{2}$ $\qquad$ $x = \frac{5}{2}$

$y = 7 - \frac{9}{2}$ $\quad$ $y = 7 - \frac{5}{2}$

$y = \frac{5}{2}$ $\quad$ $y = \frac{9}{2}$

$$\therefore\ x = \frac{9}{2}, y = \frac{5}{2} \quad or\ x = \frac{5}{2}, y = \frac{9}{2}$$

b) $log_2\, x - log_4(3x+4) = 0$

$$log_a b = \frac{log_c b}{log_c a}$$

$$\log b^n = n\ log\ b$$

$$log_a a = 1$$

$$log_2\, x - \frac{log_2(3x+4)}{log_2 4} = 0$$

$$log_2\, x - \frac{log_2(3x+4)}{log_2 2^2} = 0$$

$$log_2\, x - \frac{log_2(3x+4)}{2log_2 2} = 0$$

$$log_2\, x - \frac{log_2(3x+4)}{2(1)} = 0$$

$$log_2\, x - \frac{1}{2}log_2(3x+4) = 0$$

$$log_2\, x = \frac{1}{2}log_2(3x+4)$$

$$log_2\, x = log_2(3x+4)^{\frac{1}{2}}$$

$$x = (3x+4)^{\frac{1}{2}}$$

$$x^2 = 3x + 4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4 \quad or \quad x = -1$$

$$Since\ x > 0, \therefore x = 4$$

8. (a) Solve the following equation $\left|\frac{3}{x-4}\right| = 7, x \neq 4$.

(b) Find the solution set for the inequality, $\frac{-4-x}{x-3} \geq x+4,\ x \neq 3$.

SOLUTION

(a) $\left|\frac{3}{x-4}\right| = 7$

$\frac{3}{x-4} = 7$	or	$\frac{3}{x-4} = -7$
$3 = 7(x-4)$		$3 = -7(x-4)$
$3 = 7x - 28$		$3 = -7x + 28$
$7x = 3 + 28$		$7x = 28 - 3$
$7x = 31$		$7x = 25$
$x = \frac{31}{7}$		$x = \frac{25}{7}$

(b) $\frac{-4-x}{x-3} \geq x+4, \quad x \neq 3$

$$\frac{-4-x}{x-3} \geq x+4$$

$$\frac{-4-x}{x-3} - x - 4 \geq 0$$

$$\frac{-4-x-x(x-3)-4(x-3)}{x-3} \geq 0$$

$$\frac{-4-x-x^2+3x-4x+12}{x-3} \geq 0$$

$$\frac{-x^2-2x+8}{x-3} \geq 0$$

$$\frac{-(x^2+2x-8)}{x-3} \geq 0$$

$$\frac{(x^2+2x-8)}{x-3} \leq 0$$

$$\frac{(x+4)(x-2)}{x-3} \leq 0$$

$(x+4)=0$	$(x-2)=0$	$(x-3)=0$
$x=-4$	$x=2$	$x=3$

	$(-\infty,-4)$	$(-4,2)$	$(2,3)$	$(3,\infty)$
$(x+4)$	-	+	+	+
$(x-2)$	-	-	+	+
$(x-3)$	-	-	-	+
$\frac{(x+4)(x-2)}{x-3}$	⊖	+	⊖	+

$\therefore$ *Solution set*; $\{x: -\infty < x \leq -4 \;\cup\; 2 \leq x < 3\}$

9. Given $f(x) = e^x$ and $g(x) = |x-3|$.

(a) Show that $(f \circ g)(x) = \begin{cases} e^{x-3}, & x \geq 3 \\ e^{-(x-3)}, & x < 3 \end{cases}$

(b) Sketch the graph of $y = (f \circ g)(x)$. Hence, state the interval in which $(f \circ g)^{-1}(x)$ exists.

(c) Determine $(f \circ g)^{-1}(x)$,for $x \geq 3$.

(d) Find the function $h(x)$ for $x > \frac{1}{3}$, given that $(h \circ f)(x) = \frac{2e^x}{1-3e^x}$. Hence, show that h(x) is a one to one function.

SOLUTION

$f(x) = e^x$ and $g(x) = |x-3|$

a) $(f \circ g)(x) = f(|x-3|)$

$$= e^{|x-3|}$$

$$|x-3| = \begin{cases} x-3, & x-3 \geq 0 \\ -(x-3), & x-3 < 0 \end{cases}$$

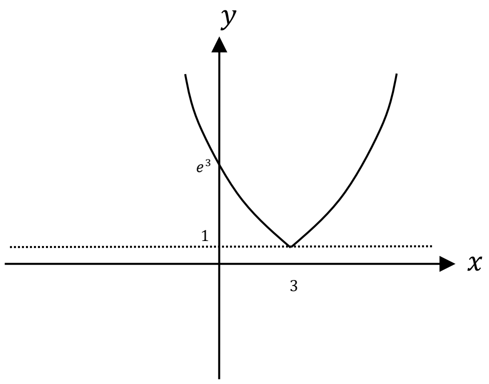
$$= \begin{cases} x-3, & x \geq 3 \\ -(x-3), & x < 3 \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$(f \circ g)(x) = e^{|x-3|}$$

$$= \begin{cases} e^{x-3}, & x \geq 3 \\ e^{-(x-3)}, & x < 3 \end{cases}$$

b)



$(f \circ g)^{-1}(x)$ exists for $x \geq 3 \quad or\; x \; < 3$

c) $(f \circ g)^{-1}(x) \; for \; x \geq 3$

$$(f \circ g)(x) = e^{x-3}$$

$$y = e^{x-3}$$

$$\ln y = \ln e^{x-3}$$

$$\ln y = x - 3$$

$$x = \ln y + 3$$

$$\therefore \; (f \circ g)^{-1}(x) = \ln x + 3$$

d) $f(x) = e^x,$

$$(h \circ f)(x) = \frac{2e^x}{1-3e^x}$$

$$h[f(x)] = \frac{2e^x}{1-3e^x}$$

$$h[e^x] = \frac{2e^x}{1-3e^x}$$

$$Let \; y = e^x$$

$$h[y] = \frac{2y}{1-3y}$$

$$h[x] = \frac{2x}{1-3x}$$

Show that h(x) is a one to one function

$$h(x) = \frac{2x}{1-3x}$$

Let $h(x_1) = h(x_2)$

$$\frac{2x_1}{1-3x_1} = \frac{2x_2}{1-3x_2}$$

$$2x_1(1-3x_2) = 2x_2(1-3x_1)$$

$$x_1(1-3x_2) = x_2(1-3x_1)$$

$$x_1 - 3x_1x_2 = x_2 - 3x_1x_2$$

$$x_1 = x_2$$

$Since\ x_1 = x_2, therefore\ h(x) is\ one\ to\ one\ function$

10. (a) Given $P = \begin{bmatrix} 2 & 0 & -4 \\ -1 & 6 & 2 \end{bmatrix}, Q = \begin{bmatrix} -1 & 3 \\ 0 & 2 \\ -6 & 5 \end{bmatrix}$ $and\ R = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 3 & 4 & 1 \end{bmatrix}$.

Find R^{-1} by using the elementary row operations method.

Hence, if $RX = 3Q + P^T$, determine the matrix X.

(b) Ahmad bought an examination pad, 2 pens and a tube of liquid paper for RM18. Ali spent RM24 for 3 examination pads, 2 pens and 2 tubes of liquid paper. In the meantime Abu spent RM36 at the same store for 3 examination pads, 4 pens and a tube of liquid paper. Let x, y and z represent the price per unit for examination pad, pen and tube of liquid paper respectively.

i. Obatain the system of linear equations from the above information.
ii. Write the system in the form of matrix equation $AX = B$.
iii. State the price of each unit of examination pad, pen and tube of liquid paper.
iv. Aminah bought 4 examination pads, 5 pens and 1 tube of liquid paper. What is the total price paid?

SOLUTION

a) $P = \begin{bmatrix} 2 & 0 & -4 \\ -1 & 6 & 2 \end{bmatrix}, Q = \begin{bmatrix} -1 & 3 \\ 0 & 2 \\ -6 & 5 \end{bmatrix}$ $and\ R = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 2 & 2 & 0 & 1 & 0 \\ 3 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2^* = 3R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 4 & 1 & 3 & -1 & 0 \\ 3 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3^* = 3R_1 - R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 4 & 1 & 3 & -1 & 0 \\ 0 & 2 & 2 & 3 & 0 & -1 \end{array}\right)$$

$$\xrightarrow{R_2^* = \frac{1}{4}R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 2 & -2 & 3 & 0 & -1 \end{array}\right)$$

$$\xrightarrow{R_3^* = 2R_2 - R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & -\frac{3}{2} & -\frac{3}{2} & -\frac{1}{2} & 1 \end{array}\right)$$

$$\xrightarrow{R_3^* = -\frac{2}{3}R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & 1 & 1 & \frac{1}{3} & -\frac{2}{3} \end{array}\right)$$

$$\xrightarrow{R_2^* = R_2 - \frac{1}{4}R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 1 & 1 & \frac{1}{3} & -\frac{2}{3} \end{array}\right)$$

$$\xrightarrow{R_1^* = R_1 - R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 1 & 1 & \frac{1}{3} & -\frac{2}{3} \end{array}\right)$$

$$\xrightarrow{R_1^* = R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 1 & 1 & \frac{1}{3} & -\frac{2}{3} \end{array}\right)$$

$$R^{-1} = \begin{bmatrix} -1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6} \\ 1 & \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

$$RX = 3Q + P^T$$

$$R^{-1}RX = R^{-1}(3Q + P^T)$$

$$X = R^{-1}(3Q + P^T)$$

$$=\begin{bmatrix}-1 & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6}\\ 1 & \frac{1}{3} & -\frac{2}{3}\end{bmatrix}\left[3\begin{bmatrix}-1 & 3\\ 0 & 2\\ -6 & 5\end{bmatrix}+\left(\begin{bmatrix}2 & 0 & -4\\ -1 & 6 & 2\end{bmatrix}\right)^T\right]$$

$$=\begin{bmatrix}-1 & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6}\\ 1 & \frac{1}{3} & -\frac{2}{3}\end{bmatrix}\left[\begin{bmatrix}-3 & 9\\ 0 & 6\\ -18 & 15\end{bmatrix}+\begin{bmatrix}2 & -1\\ 0 & 6\\ -4 & 2\end{bmatrix}\right]$$

$$=\begin{bmatrix}-1 & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6}\\ 1 & \frac{1}{3} & -\frac{2}{3}\end{bmatrix}\begin{bmatrix}-1 & 8\\ 0 & 12\\ -22 & 17\end{bmatrix}$$

$$=\begin{bmatrix}(-1)(-1)+\left(\frac{1}{3}\right)(0)+\left(\frac{1}{3}\right)(-22) & (-1)(8)+\left(\frac{1}{3}\right)(12)+\left(\frac{1}{3}\right)(17)\\ \left(\frac{1}{2}\right)(-1)+\left(-\frac{1}{3}\right)(0)+\left(\frac{1}{6}\right)(-22) & \left(\frac{1}{2}\right)(8)+\left(-\frac{1}{3}\right)(12)+\left(\frac{1}{6}\right)(17)\\ (1)(-1)+\left(\frac{1}{3}\right)(0)+\left(-\frac{2}{3}\right)(-22) & (1)(8)+\left(\frac{1}{3}\right)(12)+\left(-\frac{2}{3}\right)(17)\end{bmatrix}$$

$$=\begin{bmatrix}1+0-\frac{22}{3} & -8+\frac{12}{3}+\frac{17}{3}\\ -\frac{1}{2}+0-\frac{22}{6} & 4-4+\frac{17}{6}\\ -1+0+\frac{44}{3} & 8+4-\frac{34}{3}\end{bmatrix}$$

$$=\begin{bmatrix}-\frac{19}{3} & \frac{5}{3}\\ -\frac{25}{6} & \frac{17}{6}\\ \frac{41}{3} & \frac{2}{3}\end{bmatrix}$$

bi) $x+2y+z=18$

$3x+2y+2z=24$

$3x+4y+z=36$

bii) $$\begin{bmatrix}1 & 2 & 1\\3 & 2 & 2\\3 & 4 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}18\\24\\36\end{bmatrix}$$

biii) $$AX=B$$

$$A^{-1}AX=A^{-1}B$$

$$X=A^{-1}B$$

$$=\begin{bmatrix}-1 & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{2} & -\frac{1}{3} & \frac{1}{6}\\ 1 & \frac{1}{3} & -\frac{2}{3}\end{bmatrix}\begin{bmatrix}18\\24\\36\end{bmatrix}$$

$$=\begin{bmatrix}(-1)(18)+\left(\frac{1}{3}\right)(24)+\left(\frac{1}{3}\right)(36)\\ \left(\frac{1}{2}\right)(18)+\left(-\frac{1}{3}\right)(24)+\left(\frac{1}{6}\right)(36)\\ (1)(18)+\left(\frac{1}{3}\right)(24)+\left(-\frac{2}{3}\right)(36)\end{bmatrix}$$

$$=\begin{bmatrix}-18+8+12\\9-8+6\\18+8-24\end{bmatrix}$$

$$=\begin{bmatrix}2\\7\\2\end{bmatrix}$$

$$\boldsymbol{Exam\ pad=RM2,\quad Pen=RM7,\quad Liquid\ paper=RM2}$$

biv) $x-Exampad;\quad y-Pen;\quad z-Liquid\ Paper$

$$Total\ price=4(2)+5(7)+1(2)$$

$$=8+35+2$$

$$=RM45.00$$