

SULIT
QS015/2
Mathematics
Paper 2
Semester 1
Session 2016/2017
2 hours

QS015/2
Matematik
Kertas 2
Semester I
Sesi 2016/2017
2 jam



**KEMENTERIAN
PENDIDIKAN
MALAYSIA**

BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi 11 halaman bercetak.

This question paper consists of 11 printed pages.

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

ARAHAN KEPADA CALON:

*Kertas soalan ini mengandungi **10** soalan.*

*Jawab **semua** soalan.*

Semua jawapan hendaklah ditulis pada buku jawapan yang disediakan. Gunakan muka surat baru bagi nombor soalan yang berbeza.

Markah penuh yang diperuntukkan bagi setiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja yang boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π , e , surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Trigonometry
Trigonometri

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Differentiation
Pembezaan

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{d^2 y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

Sphere
Sfera

$$V = \frac{4}{3} \pi r^3$$

$$S = 4 \pi r^2$$

Right circular cone
Kon membulat tegak

$$V = \frac{1}{3} \pi r^2 h$$

$$S = \pi r^2 + \pi r h$$

Right circular cylinder
Silinder membulat tegak

$$V = \pi r^2 h$$

$$S = 2\pi r^2 + 2\pi r h$$

1 Express

Ungkap

$$\frac{x^2}{x^2 - 2x - 3}$$

in partial fractions form.

dalam bentuk pecahan separa.

[5 marks]

[5 markah]

2 Evaluate the following limits, if exist.

Nilaikan had berikut, jika wujud.

(a) $\lim_{x \rightarrow 2} \frac{x-2}{x^4-16}$.

[3 marks]

[3 markah]

(b) $\lim_{x \rightarrow \infty} \frac{(2-x)(x-1)}{(x-3)^2}$.

[3 marks]

[3 markah]

3 Show that

Tunjukkan bahawa

$$\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x.$$

Hence, solve $\frac{\sin^2 x}{1 - \cos x} = \cos 2x$ for $0^\circ \leq x \leq 360^\circ$.

Seterusnya, selesaikan $\frac{\sin^2 x}{1 - \cos x} = \cos 2x$ untuk $0^\circ \leq x \leq 360^\circ$.

[7 marks]

[7 markah]

4 Consider a function

Pertimbangkan fungsi

$$f(x) = \frac{1}{2 - \sqrt{x}}.$$

(a) Find $\lim_{x \rightarrow \infty} f(x)$ and state the equation of horizontal asymptote for f .

Cari $\lim_{x \rightarrow \infty} f(x)$ dan nyatakan persamaan asimptot mengufuk bagi f .

[3 marks]

[3 markah]

(b) By using the first principal of derivative, find $f'(x)$.

Dengan menggunakan prinsip pertama terbitan, cari $f'(x)$.

[4 marks]

[4 markah]

- 5 (a) Use the derivative to find the maximum area of a rectangle that can be inscribed in a semicircle of radius 10 cm.

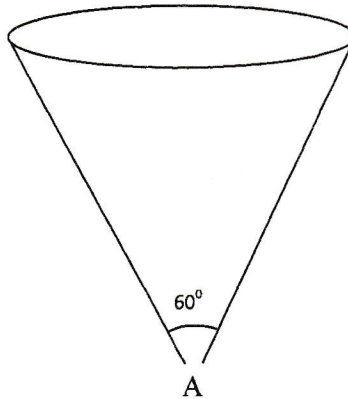
Gunakan terbitan untuk mencari luas maksima suatu segi empat tepat yang boleh diterap dalam semibulatan berjejari 10 cm.

[5 marks]

[5 markah]

- (b) A cone-shaped tank as shown below.

Sebuah tangki berbentuk kon seperti ditunjukkan di bawah.



Water flows through a hole A at rate of 6 cm^3 per second. Find the rate of change in height of the water when the volume of water in the cone is $24\pi \text{ cm}^3$.

Air mengalir keluar melalui lubang A pada kadar 6 cm^3 per saat. Cari kadar perubahan tinggi air apabila isipadu air dalam kon ialah $24\pi \text{ cm}^3$.

[6 marks]

[6 markah]

- 6 (a) Polynomial $P(x)$ has a remainder 3 when divided by $(x+3)$. Find the remainder of $P(x)+2$ when divided by $(x+3)$.

Polinomial $P(x)$ berbaki 3 apabila dibahagi dengan $(x+3)$. Cari baki apabila polinomial $P(x)+2$ dibahagi dengan $(x+3)$.

[3 marks]

[3 markah]

- (b) Polynomial $P_1(x) = x^3 + ax^2 - 5bx - 7$ has a factor $(x-1)$ and remainder R_1 when divided by $(x+1)$, while a polynomial $P_2(x) = x^3 - ax^2 + bx + 6$ has a remainder R_2 when divided by $(x-1)$. Find the value of the constants a and b if $R_1 + R_2 = 5$. Hence, obtain the zeroes for $P_1(x)$.

Polinomial $P_1(x) = x^3 + ax^2 - 5bx - 7$ mempunyai faktor $(x-1)$ dan berbaki

R_1 apabila dibahagi dengan $(x+1)$, manakala polinomial

$P_2(x) = x^3 - ax^2 + bx + 6$ mempunyai baki R_2 apabila dibahagi dengan

$(x-1)$. Cari nilai pemalar a dan b jika $R_1 + R_2 = 5$.

Seterusnya, dapatkan pensifar bagi $P_1(x)$.

[9 marks]

[9 markah]

7 Consider a function

Pertimbangkan fungsi

$$f(x) = \sqrt{3} \cos 2x + 2 \sin 2x.$$

- (a) Express f in the form of $R \cos(2x - \alpha)$ for $R > 0$, $0^\circ \leq x \leq 90^\circ$ and α to the nearest minute. State the maximum and minimum values of f .

Ungkap f dalam bentuk $R \cos(2x - \alpha)$ bagi $R > 0$, $0^\circ \leq x \leq 90^\circ$ dan α dalam minit terhampir. Nyatakan nilai maksimum dan minimum untuk f .

[7 marks]

[7 markah]

- (b) Hence, solve $\sqrt{3} \cos 2x + 2 \sin 2x = -\sqrt{2}$ for $0^\circ \leq x < 180^\circ$. Give your answer to the nearest minute.

Seterusnya, selesaikan $\sqrt{3} \cos 2x + 2 \sin 2x = -\sqrt{2}$ bagi $0^\circ \leq x < 180^\circ$. Beri jawapan anda dalam minit terhampir.

[5 marks]

[5 markah]

8 The parametric equations of a curve is given by

Persamaan berparameter suatu lengkung diberi oleh

$$x = e^{2t+1}, \quad y = e^{-(2t-1)}.$$

- (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $t = 1$.

Cari $\frac{dy}{dx}$ dan $\frac{d^2y}{dx^2}$ apabila $t = 1$.

[7 marks]

[7 markah]

- (b) Given $z = x^2 - xy$. Express z in terms of t and find $\frac{dz}{dt}$.

Hence, deduce the set value of t such that $\frac{dz}{dt}$ is positive.

Diberi $z = x^2 - xy$. Ungkapkan z dalam sebutan t dan cari $\frac{dz}{dt}$.

Seterusnya, deduksikan set nilai t supaya $\frac{dz}{dt}$ adalah positif.

[5 marks]

[5 markah]

- 9 (a) Given $f(x) = \frac{2|x|}{x} + 5x$. Compute $\lim_{x \rightarrow 0^+} f(x)$ and $\lim_{x \rightarrow 0^-} f(x)$.

Is f continuous at $x = 0$? Give your reason.

Diberi $f(x) = \frac{2|x|}{x} + 5x$. Hitung $\lim_{x \rightarrow 0^-} f(x)$ dan $\lim_{x \rightarrow 0^+} f(x)$.

Adakah f selanjar pada $x = 0$? Beri alasan anda.

[5 marks]

[5 markah]

- (b) The continuous function g is defined by

Fungsi selanjar g ditakrifkan sebagai

$$g(x) = \begin{cases} \sqrt{5-x}, & x < a, \\ 3x-1, & x \geq a. \end{cases}$$

Find the value of a .

Cari nilai a .

[8 marks]

[8 markah]

- 10 By writing $\tan x$ in terms of $\sin x$ and $\cos x$, show that

Dengan menulis $\tan x$ dalam sebutan $\sin x$ dan $\cos x$, tunjukkan bahawa

$$\frac{d}{dx}(\tan x) = \sec^2 x.$$

[3 marks]

[3 markah]

- (a) If $y = \tan x$, find $\frac{d^2y}{dx^2}$ in terms of y . Hence, determine the range of value of x such that $\frac{d^2y}{dx^2} > 0$ for $0 < x < \pi$.

Jika $y = \tan x$, cari $\frac{d^2y}{dx^2}$ dalam sebutan y . Seterusnya, tentukan julat nilai x

supaya $\frac{d^2y}{dx^2} > 0$ bagi $0 < x < \pi$.

[7 marks]

[7 markah]

- (b) If $y = \tan(x + y)$, find $\frac{dy}{dx}$ in terms of x and y . Hence, show that

$$\frac{dy}{dx} = \operatorname{cosec}^2 2\alpha \text{ when } x = y = \alpha.$$

Jika $y = \tan(x + y)$, cari $\frac{dy}{dx}$ dalam sebutan x dan y . Seterusnya, tunjukkan

bahawa $\frac{dy}{dx} = \operatorname{cosec}^2 2\alpha$ apabila $x = y = \alpha$.

[5 marks]

[5 markah]