SULIT
QS025/1
Mathematics
Paper 1
Semester II
Session 2016/2017
2 hours

QS025/1
Matematik
Kertas 1
Semester II
Sesi 2016/2017
2 jam

KEMENTERIAN PENDIDIKAN MALAYSIA
MINISTRY OF EDUCATION MALAYSIA

BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

CHOW CHOON WOOI

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

ARAHAN KEPADA CALON:

Kertas soalan ini mengandungi **10** *soalan.*

Jawab **semua** *soalan.*

Semua jawapan hendaklah ditulis pada buku jawapan yang disediakan. Gunakan muka surat baharu bagi nombor soalan yang berbeza.

Markah penuh yang diperuntukkan bagi setiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja yang boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π, *e, surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.*

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Trigonometry
Trigonometri

$$\sin\left(A \pm B\right) = \sin A \cos B \pm \cos A \sin B$$

$$\cos\left(A \pm B\right) = \cos A \cos B \mp \sin A \sin B$$

$$\tan\left(A \pm B\right) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Differentiation and Integration
Pembezaan dan Pengamiran

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\cos ec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cos ec\, x) = -\cos ec\, x \cot x$$

$$\int f'(x)e^{f(x)}\, dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$$

$$\int f'(x)[f(x)]^n\, dx = \frac{[f(x)]^{n+1}}{n+1} + c, \ n \neq -1$$

$$\int u\, dv = uv - \int v\, du$$

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LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Numerical Methods
Kaedah Berangka

Newton-Raphson Method:
Kaedah Newton-Raphson:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots$$

Trapezoidal Rule:
Petua Trapezium:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})], \quad h = \frac{b-a}{n}$$

Conics
Keratan Kon

Circle:
Bulatan:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$r = \sqrt{f^2 + g^2 - c}$$

$$d = \frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$$

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Parabola:
***Parabola*:**

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

$$F(h+p,k)$$

$$F(h,k+p)$$

Ellipse:
***Elips*:**

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$F(h\pm c,k)$$

$$F(h,k\pm c)$$

Vectors
Vektor

Line and Plane:
***Garis dan Satah*:**

$$\vec{r} = \vec{a} + t\vec{v}$$

$$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

1 Find the angle between the line $\ell : \langle x, y, z \rangle = \langle 1, 3, -1 \rangle + t \langle 2, 1, 0 \rangle$ and

the plane $\Pi : 3x - 2y + z = 5$.

Cari sudut di antara garis $\ell : \langle x, y, z \rangle = \langle 1, 3, -1 \rangle + t \langle 2, 1, 0 \rangle$ dan

satah $\Pi : 3x - 2y + z = 5$.

[*6 marks*]

[*6 markah*]

2 Solve

Selesaikan

$$\int \frac{e^{2x}}{\left(1 - e^{2x}\right)} \, dx \,.$$

[*6 marks*]

[*6 markah*]

3 Given four points $A = (-2, -8, 4)$, $B = (2, -\omega, -1)$, $C = (0, -9, 0)$ and $D = (-4, -3, 7)$.

Determine the value of ω if $\overrightarrow{AB} \cdot \left(\overrightarrow{AC} \times \overrightarrow{AD} \right) = 64$.

Diberi empat titik $A = (-2, -8, 4)$, $B = (2, -\omega, -1)$, $C = (0, -9, 0)$ dan $D = (-4, -3, 7)$.

Tentukan nilai ω jika $\overrightarrow{AB} \cdot \left(\overrightarrow{AC} \times \overrightarrow{AD} \right) = 64$.

[*7 marks*]

[*7 markah*]

4   Find the vertex, focus and directrix for the parabola $y^2 + 64 = 8y - 16x$.

Hence, sketch and label the vertex, focus and directrix for the curve.

Cari bucu, fokus dan direktriks bagi parabola $y^2 + 64 = 8y - 16x$.

Seterusnya, lakar dan labelkan bucu, fokus dan direktriks bagi lengkung tersebut.

[7 *marks*]

[7 *markah*]

5   The end points of the diameter of a circle are $P(0,1)$ and $Q(3,-3)$.

Titik hujung garis pusat suatu bulatan adalah $P(0,1)$ dan $Q(3,-3)$.

(a)   Determine an equation of the circle.

Tentukan persamaan bulatan tersebut.

[5 *marks*]

[5 *markah*]

(b)   Find an equation of the tangent line to the circle at the point $P(0,1)$.

Cari persamaan garis tangen kepada bulatan tersebut di titik $P(0,1)$.

[5 *marks*]

[5 *markah*]

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6   In a Chemistry experiment, sodium hydroxide, NaOH, reacts with hydrochloric acid, HCl, to form sodium chloride salt, NaCl, and water. Before the reaction starts, no NaCl salt is formed. At time t (minute), the mass of NaCl salt formed is x grams and the rate of change of x is given by $\dfrac{dx}{dt} = \alpha(50 - x)$, where α is a positive constant.

Dalam suatu eksperimen Kimia, natrium hidroksida, NaOH, bertindak balas dengan asid hidroklorik, HCl, untuk membentuk garam natrium klorida, NaCl, dan air. Sebelum tindak balas bermula, tiada garam NaCl terbentuk. Pada masa t (minit), jisim garam NaCl yang terbentuk adalah x gram dan kadar perubahan x diberi oleh $\dfrac{dx}{dt} = \alpha(50 - x)$, dengan α adalah suatu pemalar positif.

(a)   Find the general solution for the above equation.

Dapatkan penyelesaian am bagi persamaan di atas.

[5 marks]

[5 markah]

(b)   Find the particular solution if 35 grams of NaCl salt has formed in the first 30 minutes.

Dapatkan penyelesaian khusus jika 35 gram garam NaCl telah terbentuk dalam tempoh 30 minit pertama.

[3 marks]

[3 markah]

(c)   Hence, find

Seterusnya, cari

      (i)   the mass of NaCl salt formed in 60 minutes.

          jisim garam NaCl yang terbentuk dalam masa 60 minit.

[2 marks]

[2 markah]

(ii)   the time taken to form 40 grams of NaCl salt.

masa yang diambil untuk membentuk 40 gram garam NaCl.

[2 *marks*]

[2 *markah*]

7   (a)   Show that the equation $-4x^2 + 5x + 7 = 0$ has a root on the interval $[-2, 0]$.

Use the Newton-Raphson method to find the root of the equation correct to four decimal places.

Tunjukkan bahawa persamaan $-4x^2 + 5x + 7 = 0$ mempunyai satu punca pada selang $[-2, 0]$. Gunakan kaedah Newton-Raphson untuk mencari punca kepada persamaan tersebut betul hingga empat tempat perpuluhan.

[7 *marks*]

[7 *markah*]

(b)   Estimate the value of $\int_{-\pi}^{0} x \cos x \, dx$ using trapezoidal rule with subinterval $\dfrac{\pi}{4}$.

Give your answer correct to four decimal places.

Anggarkan nilai bagi $\int_{-\pi}^{0} x \cos x \, dx$ menggunakan petua trapezium dengan subselang $\dfrac{\pi}{4}$. Berikan jawapan anda betul hingga empat tempat perpuluhan.

[5 *marks*]

[5 *markah*]

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8   Given the curve $y = 4x^2$ and the line $y = 6x$.

Diberi lengkung $y = 4x^2$ dan garis $y = 6x$.

(a)   Find the intersection points.

Cari titik persilangan.

[2 *marks*]

[2 *markah*]

(b)   Sketch the region enclosed by the curve and the line.

Lakar rantau yang dibatasi oleh lengkung dan garis tersebut.

[3 *marks*]

[3 *markah*]

(c)   Calculate the area of the region enclosed by the curve and the line.

Hitung luas rantau yang dibatasi oleh lengkung dan garis tersebut.

[3 *marks*]

[3 *markah*]

(d)   Calculate the volume of the solid generated when the region is revolved completely about the y-axis.

Hitung isipadu bongkah yang terjana apabila rantau tersebut dikisar sepenuhnya pada paksi-y.

[4 *marks*]

[4 *markah*]

9   (a)   If the line $\ell_1 : \langle x, y, z \rangle = \langle 1,1,2 \rangle + t \langle 2,-1,3 \rangle$ does not intersect with the plane

$\Pi_1 : Ax + By + Cz = 0$, show that $2A - B + 3C = 0$.

Hence, find the equation of plane Π_1 if the plane passes through

the point $(1,0,1)$.

Jika garis $\ell_1 : \langle x, y, z \rangle = \langle 1,1,2 \rangle + t \langle 2,-1,3 \rangle$ tidak bersilang dengan satah

$\Pi_1 : Ax + By + Cz = 0$, tunjukkan bahawa $2A - B + 3C = 0$.

Seterusnya, cari persamaan satah Π_1 jika satah tersebut melalui titik $(1,0,1)$.

[6 marks]

[6 markah]

    (b)   Given the line $\ell_2 : x = x_0 + tv_1,\ y = y_0 + tv_2,\ z = z_0 + tv_3$, the plane

$\Pi_2 : x - y + 2z = 0$ and a point $(x_0, y_0, z_0) \neq (0,0,0)$ is on the plane.

Diberi garis $\ell_2 : x = x_0 + tv_1,\ y = y_0 + tv_2,\ z = z_0 + tv_3$, satah

$\Pi_2 : x - y + 2z = 0$ dan satu titik $(x_0, y_0, z_0) \neq (0,0,0)$ berada di atas satah

tersebut.

       (i)   If ℓ_2 is perpendicular to the plane Π_2, show that

$\langle v_1, v_2, v_3 \rangle = v_2 \langle -1,1,-2 \rangle;\ v_2 \neq 0$.

Jika ℓ_2 berserenjang dengan satah Π_2, tunjukkan bahawa

$\langle v_1, v_2, v_3 \rangle = v_2 \langle -1,1,-2 \rangle;\ v_2 \neq 0$.

[4 marks]

[4 markah]

       (ii)   Give one example of the equation of straight line which satisfy

part 9(b)(i).

Berikan satu contoh persamaan garis lurus yang menepati

bahagian 9(b)(i).

[3 marks]

[3 markah]

10   (a)   Show that the expression $\dfrac{4x^4 + 2x^2 - 1}{(2x-3)^2(x+1)}$ can be written as

$$x + 2 + \frac{A}{2x-3} + \frac{B}{(2x-3)^2} + \frac{C}{x+1}.$$

Tunjukkan bahawa ungkapan $\dfrac{4x^4 + 2x^2 - 1}{(2x-3)^2(x+1)}$ *boleh ditulis sebagai*

$$x + 2 + \frac{A}{2x-3} + \frac{B}{(2x-3)^2} + \frac{C}{x+1}.$$

[3 *marks*]

[3 *markah*]

(b)   From part **10**(a), determine the values of A, B and C.

Hence, solve $\displaystyle\int \frac{4x^4 + 2x^2 - 1}{(2x-3)^2(x+1)}\, dx.$

Daripada bahagian **10**(a), *tentukan nilai* A, B *dan* C.

Seterusnya, selesaikan $\displaystyle\int \frac{4x^4 + 2x^2 - 1}{(2x-3)^2(x+1)}\, dx.$

[12 *marks*]

[12 *markah*]

END OF QUESTION PAPER

KERTAS SOALAN TAMAT