QS026/2
Mathematics
Paper 2
Semester II
2007/2008
2 hours

QS026/2
Matematik
Kertas 2
Semester II
2007/2008
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.
This booklet consists of **15** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trapezium Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left\{(y_0 + y_n) + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right)\right\}, \text{ where } h = \frac{b - a}{n}.$$

Newton–Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1,2,3,\ldots$$

Statistics

For ungrouped data, the kth percentile,

$$P_k = \begin{cases} \dfrac{x_{(s)} + x_{(s+1)}}{2}, & \text{if } s \text{ is an integer} \\ x_{([s])}, & \text{if } s \text{ is a non-integer} \end{cases}$$

where $s = \dfrac{n \times k}{100}$ and $[\,s\,]$ = the least integer greater than k.

For grouped data, the kth percentiles, $P_k = L_k + \left[\dfrac{\left(\dfrac{k}{100}\right)n - F_{k-1}}{f_k}\right]c$.

1. Given a sample of the heights (in cm) of seedlings in an experiment as follows:

$$76.0,\ 68.1,\ 73.4,\ 80.2,\ 75.4,\ 78.3$$

(a) Find the mean and standard deviation of the above data.

[4 marks]

(b) If it was later discovered that the measuring scales were reading a cm below the correct height, state the changes (if any) on the mean and the variance.

[2 marks]

2. The probability of a woman giving birth to a baby boy is 0.6. If the woman gave birth to 3 children, find the probability that

(a) the number of sons exceeds the number of daughters.

[3 marks]

(b) all three children are of the same gender.

[3 marks]

3. The probability density function of a discrete random variable X is

$$P(X = x) = \frac{kx}{2} \quad \text{where } x = 1, 2, 3, 4, 5, 6$$

and k is a constant.

(a) Show that $k = \frac{2}{21}$.

[2 marks]

(b) Find $\text{Var}(X)$.

[4 marks]

4. The probability that a student passes Mathematics is 0.4. If the student passes Mathematics, the probability that the student will pass Physics is 0.7. If the student fails Mathematics, the probability the student will pass Physics is 0.63.

(a) Calculate the probability that the student passes

(i) Physics.

[2 marks]

(ii) Mathematics if the student passes Physics.

[2 marks]

(b) Determine whether the events of a student passing Mathematics and Physics are independent.

[2 marks]

5. A parachutist jumps off an aeroplane on a regular training session. When his parachute opens, he travels vertically downward with a velocity $v_\circ$. The velocity of the parachutist at time t minutes is v and his acceleration is given by

$$\frac{dv}{dt} = g - \alpha v$$

where g is acceleration due to gravity and α is a constant.

(a) Show that $v = \frac{g}{\alpha} - \left(\frac{g}{\alpha} - v_\circ\right)e^{-\alpha t}$.

[9 marks]

(b) Determine the difference in the velocities of the parachutist from the fifth to the tenth minutes.

[2 marks]

(c) Find the velocity of the parachutist after a very long period.

[2 marks]

6. By sketching the graph of $y = x^3$ and $y = 4x - 2$, show that the equation $x^3 - 4x + 2 = 0$ has three real roots.

[3 marks]

(a) Show that one of the real roots of the equation lies between 1 and 2.

[3 marks]

(b) By using the Newton-Raphson method and initial value 2, determine the real root that lies between 1 and 2, correct to two decimal places.

[6 marks]

7. The consultation times (in minutes) for 100 patients at a private clinic is given in the table below.

Time interval (minutes), x	**Frequency,** f
0-9	9
10-19	34
20-29	20
30-39	18
40-49	9
50-59	7
60-69	3

Given $\sum xf = 2620$ and $\sum x^2 f = 91858$.

(a) Find

(i) the mode and median.

[4 marks]

(ii) the mean standard deviation and Pearson skewness coefficient.

[5 marks]

(b) Hence, state with reason, which of the above measures of central tendency better describes the distribution of the data.

[2 marks]

8. **(a)** There are 10 men, 15 women and 12 children participating in a family day event.

(i) In how many ways can a group of 7 men, 13 women and 10 children be formed if a particular lady and a particular child must be in that group?

[3 marks]

(ii) Thirty participants are required in an event. In how many ways can this group be formed if each group must consist of at least 8 men?

[3marks]

(b) A five-digit number may be formed from the digits 1, 2, 3, 4, 5, 6, 7 and 8 with no repetition. How many

(i) five-digit numbers having values between 10000 and 50000 can be formed?

[3 marks]

(ii) five-digit even numbers having values more than 60000 can be formed?

[3 marks]

9. The time taken by a student (in hours) to study is given by a continuous variable X, with a cumulative density function of

$$F(x)=\begin{cases} 0 & \text{if} \quad x \le 0 \\ 1-k(10-x)^2 & \text{if} \quad 0 \le x \le 10 \\ 1 & \text{if} \quad x \ge 10 \end{cases}$$

where k is a constant.

(a) Determine the value of k.

[2 marks]

(b) Find $P(3 \le X < 9)$.

[2 marks]

(c) Determine the probability density function of X for $0 \le x \le 10$.

[2 marks]

(d) Find the median of X.

[3 marks]

(e) Obtain the variance of X.

[4 marks]

10. Assume that the number of e-mails received by a student daily has a Poisson distribution with a mean of 5.

(a) **(i)** Determine the probability that the student receives between 5 and 13 e-mails daily.

[2 marks]

(ii) If the probability of a student receiving not more than m e-mails in a day is 0.616, determine the value of m.

[4 marks]

(b) If 15 days are randomly chosen, find the probability that the student receives between 5 and 13 e-mails daily for a period of 9 days.

[3 marks]

(c) If 150 days are randomly chosen, use the normal approximation to find the probability that the student receives between 5 and 13 e-mails daily for less than 70 days.

[6 marks]

END OF BOOKLET