

QS026/2
Mathematics
Paper 2
Semester II
2009/2010
2 hours

QS026/2
Matematik
Kertas 2
Semester II
2009/2010
2 jam



BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

CHOW CHOON WOOI

Kertas soalan ini mengandungi **13** halaman bercetak.
This booklet consists of 13 printed pages.

© Bahagian Matrikulasi

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trapezium Rule

$$\int_a^b f(x) dx = \frac{h}{2} \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}, \text{ where } h = \frac{b - a}{n}$$

Newton-Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \dots$$

Statistics

For ungrouped data, the k th percentile,

$$P_k = \begin{cases} \frac{x_{(s)} + x_{(s+1)}}{2}, & \text{if } s \text{ is an integer} \\ x_{([s])}, & \text{if } s \text{ is a non-integer} \end{cases}$$

where $s = \frac{n \times k}{100}$ and $[s]$ = the least integer greater than k .

For grouped data, the k th percentiles, $P_k = L_k + \left[\frac{\left(\frac{k}{100}\right)n - F_{k-1}}{f_k} \right] c$

- 1 Solve the differential equation $\frac{dy}{dx} = \frac{1}{\sqrt{xy}}$ with initial condition $y(0) = 4$.

Express y in terms of x . [5 marks]

- 2 Given two events A and B with

$$P(B) = \frac{1}{3}, \quad P(A \cup B) = \frac{3}{4} \quad \text{and} \quad P(B|A) = \frac{1}{4}.$$

Find

- (a) $P(A)$. [4 marks]

- (b) $P(\bar{B}|\bar{A})$. [2 marks]

- 3 (a) How many one-, two-, three-, and four-digit numbers can be formed using the digits 4, 5, 6, and 7, when each digit can be used only once?

[3 marks]

- (b) How many of the numbers formed in part (a) are odd and greater than 600?

[4 marks]

- 4 The following table represents the probability distribution of a discrete random variable Y .

Y	-2	-1	1	3	5
$P(Y = y)$	0.1	0.3	0.4	0.1	0.1

Find

- (a) $P(|Y| > 1)$. [2 marks]

- (b) $E(Y-3)^2$ and $\text{Var}(Y-3)$. [5 marks]

5 (a) Given $g(x) = (x-1)\sqrt{x+2}$. By using the Newton-Raphson method starting at $x_0 = 1.1$, find the root of $g(x)$. [6 marks]

(b) By using trapezoidal method, obtain the approximate value of $\int_0^1 x e^{x^2} dx$ based on four subintervals, correct to four decimal places. [4 marks]

6 The waiting time for 50 customers to have their food served at a restaurant on a particular day is shown in the following table.

Time (minutes)	Number of Customer
1 – 5	4
6 – 10	9
11 – 15	15
16 – 20	11
21 – 25	6
26 – 30	3
31 – 35	2

(a) Calculate the mean, median and mode of the waiting time. [8 marks]

(b) Plot an ogive. Hence, determine the percentage of customers who have to wait beyond 23 minutes. [5 marks]

- 7 A model for the concentration of glucose solution in the bloodstream, $C = C(t)$, is given by the differential equation $\frac{dC}{dt} = r - kC$, where r is the constant rate at which glucose solution enters the bloodstream and k is a positive constant. If $C(0) = C_0$, show that the concentration at any time t is

$$C(t) = \left(C_0 - \frac{r}{k} \right) e^{-kt} + \frac{r}{k}. \quad [8 \text{ marks}]$$

After a very long period of time, the concentration of glucose is found to be 1 unit.

If $C_0 = 9$, what is the concentration of glucose at $t = \frac{2}{k}$?

[4 marks]

- 8 In a class of 15 students of which 7 are males, 5 students wear spectacles. There are 3 male students who wear spectacles. Four students are chosen at random. Find the probability that

(a) all females are chosen. [2 marks]

(b) equal number of males and females who wear spectacles are chosen. [2 marks]

(c) all males are chosen if it is known that they all do not wear spectacles. [2 marks]

(d) more students who do not wear spectacles are chosen if it is known that they are females. [3 marks]

(e) all females or students who do not wear spectacles are chosen. [3 marks]

- 9 The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{k \ln x}{x}, & 1 < x < e \\ 0, & \text{otherwise.} \end{cases}$$

Show that $k = 2$. [3 marks]

Hence,

- (a) obtain the cumulative distribution function, $F(x)$. [3 marks]
- (b) determine the 81st percentile for the distribution of X . [3 marks]
- (c) calculate $E(X)$. [4 marks]

- 10 In any large shipment of watermelons from a particular orchard, it is known that 2% are unripe. Upon arrival of a shipment at a receiving depot, random samplings with replacement are conducted.

- (a) Calculate the probability of getting at most one unripe watermelon in a sample of size 20. [4 marks]
- (b) Approximate the probability of getting one to three unripe watermelons in a sample of size 50. [5 marks]
- (c) If the sample size is 1000, approximate the probability of getting not more than eight unripe watermelons. [6 marks]

END OF BOOKLET

CHOW CHOON WOUI