QS026/2
Mathematics
Paper 2
Semester II
Session 2010/2011
2 hours

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Matematik
Kertas 2
Semester II
Sesi 2010/2011
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

CHOW CHOON WOOI

Kertas soalan ini mengandungi **21** halaman bercetak.
This booklet consists of 21 *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions in the answer booklet provided.

Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trapezium Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left\{(y_0 + y_n) + 2\,(y_1 + y_2 + \ldots + y_{n-1})\right\}, \text{ where } h = \frac{b - a}{n}$$

Newton–Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1,2,3,\ldots$$

Statistics

For ungrouped data, the kth percentile,

$$P_k = \begin{cases} \dfrac{x_{(s)} + x_{(s+1)}}{2}, & \text{if } s \text{ is an integer} \\ x_{([s])}, & \text{if } s \text{ is a non-integer} \end{cases}$$

where $s = \dfrac{n \times k}{100}$ and $[\,s\,]$ = the least integer greater than k.

For grouped data, the kth percentiles, $P_k = L_k + \left[\dfrac{\left(\dfrac{k}{100}\right)n - F_{k-1}}{f_k}\right]c.$

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1 Obtain the general solution for $\dfrac{dy}{dx} - 2y = 4e^{2x}$.

[5 *marks*]

2 The summary statistics of the length (in cm) of a sample of 50 adult insects of a certain species is as follows

$$\sum x = 45,\ \sum x^2 = 81.$$

Calculate the mean and variance. Hence, comment on the distribution of the sample based on the coefficient of variation.

[6 *marks*]

3 By using the substitution $u = \ln x$, evaluate $\displaystyle\int_2^3 \frac{\ln x}{x}\,dx$ up to five decimal places.

[2 *marks*]

If $\displaystyle\int_2^3 \frac{\ln x}{x}\,dx$ is approximated using the trapezoidal method based on five equal subintervals, compute the error.

[5 *marks*]

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4 The cumulative probability distribution function of a discrete random variable X is

$$P(X \le x) = \begin{cases} 0 & , \quad x < -1 \\ \frac{1}{6} & , \quad -1 \le x < 2 \\ \frac{2}{3} & , \quad 2 \le x < 4 \\ \frac{5}{6} & , \quad 4 \le x < 6 \\ 1 & , \quad x \ge 6 \end{cases}$$

(a) Construct the probability distribution table of X.

[3 *marks*]

(b) Calculate $\text{Var}(X)$.

[4 *marks*]

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5 The time (in minutes) used by 120 students surfing the internet to perform a certain project is given in the following relative cumulative frequency table.

Time (x), in minutes	Relative cumulative frequency
$x \le 0$	0
$x \le 20$	$\frac{3}{40}$
$x \le 40$	$\frac{19}{60}$
$x \le 60$	$\frac{2}{3}$
$x \le 80$	$\frac{53}{60}$
$x \le 100$	1

Find

(a) the median and mean.

[6 *marks*]

(b) Pearson's skewness coefficient and comment on the value obtained.

[4 *marks*]

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6 Newton's law of cooling states that hot liquid at temperature H cools at a rate $\frac{dH}{dt}$ proportional to the difference between its temperature and temperature of the surrounding environment H_0.

Show that $H = Ae^{-kt} + H_0$, where k is the cooling rate constant and A is an integral constant.

[3 *marks*]

A hot tea at 76°C is left in a room of 22°C.

(a) Find the Newton's cooling equation.

[2 *marks*]

(b) Using a container X, it is found that after 10 minutes in the room, the temperature of tea has decreased by 10°C. Determine the temperature of tea after 15 minutes in the room.

[4 *marks*]

(c) Using a different container Y, whose $k = 0.10$, determine the time taken for the tea to cool down to room temperature.

[2 *marks*]

(d) In which of the two containers, X or Y, does the tea cools down to room temperature faster?

[1 *mark*]

7 Three boxes A, B and C has identical green and red dice as shown in the following table.

Colour \ Box	A	B	C
Green	4	5	3
Red	5	6	4

(a) If all the dice in box A are arranged in a row, in how many different arrangements can this be done?

[2 *marks*]

(b) A die is randomly drawn from each of the boxes. If all dice drawn are of the same colour, in how many different ways can this be done?

[3 *marks*]

(c) Four dice are randomly drawn without replacement from box B. How many different ways can the dice be drawn such that there are equal number of red and green dice?

[3 *marks*]

(d) A die randomly drawn from box A is put into box B. Subsequently, a die drawn from box B is put into box C. Finally, a die is drawn from box C and the colour is noted. Calculate the probability that the die drawn from C is green.

[4 *marks*]

8 In a chess tournament between A and B, the probability A wins is 0.2, B wins is 0.5 and the probability of a draw is 0.3. If A and B were to meet in three games, calculate the probability that

(a) two games are draw.

[4 *marks*]

(b) A and B win alternately.

[3 *marks*]

(c) either A or B wins all the games.

[3 *marks*]

(d) B wins at least two games.

[3 *marks*]

9 A continuous random variable, Y has a probability density function

$$f(y)=\begin{cases} c(3-y^2), & 0<y<2 \\ 0, & \text{otherwise} \end{cases}$$

Show that $c=\dfrac{3}{10}$ and $E(Y)=\dfrac{3}{5}$.

[5 *marks*]

Hence,

(a) calculate $\mathrm{P}(Y>\mathrm{E}(Y))$.

[3 *marks*]

(b) determine the mode of the distribution.

[2 *marks*]

(c) show that the median, m of the distribution satisfies the equation $m^3-9m+5=0$.

[3 *marks*]

10 The distribution of the weights of all sugar sachets produced by a particular factory is assumed to be normal with mean 25 gm and standard deviation 2 gm.

Show that the probability of a randomly selected sachet weighs within 1 gm of the mean is 0.383.

[4 *marks*]

(a) If ten sachets are randomly selected, find the probability that between four and seven sachet weigh within 1 gm of the mean.

[3 *marks*]

(b) Determine the sample size, n such that the probability that none of the sachet weighs within 1 gm of the mean is 0.021.

[4 *marks*]

(c) If one hundred sachets are randomly selected, approximate the probability that less than 40 sachets weigh within 1 gm of the mean.

[4 *marks*]

END OF QUESTION BOOKLET

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