

QS015/2
Mathematics
Paper 2
Semester I
Session 2015/2016
2 hours

QS015/2
Matematik
Kertas 2
Semester I
Sesi 2015/2016
2 jam



KEMENTERIAN
PENDIDIKAN
MALAYSIA

BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

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Kertas soalan ini mengandungi **15** halaman bercetak.

This question paper consists of 15 printed pages.

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ARAHAN KEPADA CALON:

Kertas soalan ini mengandungi **10** soalan.

Jawab **semua** soalan.

Semua jawapan hendaklah ditulis pada buku jawapan yang disediakan. Gunakan muka surat baru bagi nombor soalan yang berbeza.

Markah penuh yang diperuntukkan bagi setiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja yang boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π , e , surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.

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INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

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SENARAI RUMUS MATEMATIK

Trigonometri

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

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LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

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$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

SENARAI RUMUS MATEMATIK

Pembezaan

$f(x)$	$f'(x)$
kot x	$- \text{kosek}^2 x$
sek x	$\text{sek } x \tan x$
kosek x	$- \text{kosek } x \cot x$

Jika $x = f(t)$ dan $y = g(t)$ maka $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2 y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

Sfera

$$V = \frac{4}{3} \pi r^3$$

$$S = 4 \pi r^2$$

Kon membulat tegak

$$V = \frac{1}{3} \pi r^2 h$$

$$S = \pi r s$$

Silinder membulat tegak

$$V = \pi r^2 h$$

$$S = 2 \pi r h$$

LIST OF MATHEMATICAL FORMULAE

Differentiation

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

If $x = f(t)$ and $y = g(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

Sphere	$V = \frac{4}{3} \pi r^3$	$S = 4 \pi r^2$
Right circular cone	$V = \frac{1}{3} \pi r^2 h$	$S = \pi r s$
Right circular cylinder	$V = \pi r^2 h$	$S = 2 \pi r h$

- 1 Ungkapkan $\frac{5x^2 + 4x + 4}{(x^2 - 4)(x + 2)}$ dalam bentuk pecahan separa.

[6 markah]

- 2 Nilaikan yang berikut (jika wujud):

(a) $\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{|x - 2|}$.

[4 markah]

(b) $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{1 - x}$.

[3 markah]

- 3 Cari terbitan untuk fungsi berikut:

(a) $f(x) = \cot \sqrt{4x^2 + 1}$.

[3 markah]

(b) $f(x) = e^{2x} \ln(3x + 4)$.

[4 markah]

- 4 Diberi $\operatorname{cosec}^2 x - \cot x = 3$, tunjukkan bahawa $\cot^2 x - \cot x - 2 = 0$.

Seterusnya, selesaikan persamaan $\operatorname{cosec}^2 x - \cot x = 3$ untuk $0 \leq x \leq \pi$.

[6 markah]

1 Express $\frac{5x^2 + 4x + 4}{(x^2 - 4)(x + 2)}$ in the form of partial fractions.

[6 marks]

2 Evaluate the following (if exist):

(a) $\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{|x - 2|}$.

[4 marks]

(b) $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{1 - x}$.

[3 marks]

3 Find the derivative of the following functions:

(a) $f(x) = \cot \sqrt{4x^2 + 1}$.

[3 marks]

(b) $f(x) = e^{2x} \ln(3x + 4)$.

[4 markah]

4 Given $\operatorname{cosec}^2 x - \cot x = 3$, show that $\cot^2 x - \cot x - 2 = 0$.

Hence, solve the equation $\operatorname{cosec}^2 x - \cot x = 3$ for $0 \leq x \leq \pi$.

[6 marks]

- 5 Polinomial $P(x) = 2x^4 + ax^3 + bx^2 - 17x + c$ dengan a , b dan c adalah pemalar, mempunyai faktor $(x+2)$ dan $(x-1)$. Apabila $P(x)$ dibahagikan dengan $(x+1)$, bakinya adalah 8. Cari nilai bagi a , b dan c . Seterusnya, faktorkan $P(x)$ selengkapnya dan nyatakan pensifarnya.

[9 markah]

- 6 (a) Nilaikan had $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 3x}}{5x + 1}$.

[4 markah]

(b) Diberi $f(x) = \begin{cases} 5 - px, & -2 < x \leq -1 \\ x^2 + px + q, & -1 < x \leq 2 \\ \frac{x^2 - 4}{x - 2}, & x > 2. \end{cases}$

- (i) Cari nilai p dan q jika fungsi $f(x)$ adalah selanjar untuk semua nilai nyata bagi x .

[6 markah]

- (ii) Lakarkan graf $f(x)$ menggunakan nilai p dan q yang diperoleh dalam bahagian (i).

[3 markah]

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- 5 A polynomial $P(x) = 2x^4 + ax^3 + bx^2 - 17x + c$ where a , b and c are constants, has factors $(x+2)$ and $(x-1)$. When $P(x)$ is divided by $(x+1)$, the remainder is 8. Find the values of a , b and c . Hence, factorize $P(x)$ completely and state its zeroes.

[9 marks]

- 6 (a) Evaluate $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 3x}}{5x+1}$.

[4 marks]

(b) Given $f(x) = \begin{cases} 5 - px, & -2 < x \leq -1 \\ x^2 + px + q, & -1 < x \leq 2 \\ \frac{x^2 - 4}{x - 2}, & x > 2. \end{cases}$

- (i) Find the values of p and q if $f(x)$ is continuous for all real values of x .

[6 marks]

- (ii) Sketch the graph of $f(x)$ using the values p and q obtained in part (i).

[3 marks]

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7 Suatu lengkung diberi oleh persamaan berparameter

$$x = t - \frac{1}{t}, \quad y = t + \frac{1}{t}.$$

(a) Cari $\frac{dy}{dx}$ dan $\frac{d^2y}{dx^2}$ dalam sebutan t .

[7 markah]

(b) Dapatkan koordinat titik pegun bagi lengkung tersebut dan tentukan sifat titik tersebut.

[6 markah]

8 (a) Jika $y^2 - 2y\sqrt{1+x^2} + x^2 = 0$, tunjukkan bahawa $\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$.

[6 markah]

(b) Air mengalir dengan kadar tetap $36\pi \text{ cm}^3\text{s}^{-1}$ ke dalam suatu kon membulat tegak yang terbalik dengan sudut separa menegak 45° .

(i) Cari kadar peningkatan kedalaman air apabila ketinggian air adalah 3 cm.

[4 markah]

(ii) Cari masa yang diambil apabila ketinggian air adalah 18 cm.

[3 markah]

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7 A curve is given by the parametric equations

$$x = t - \frac{1}{t}, \quad y = t + \frac{1}{t}.$$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t .

[7 marks]

(b) Obtain the coordinates of the stationary points of the curve and determine the nature of the points.

[6 marks]

8 (a) If $y^2 - 2y\sqrt{1+x^2} + x^2 = 0$, show that $\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$.

[6 marks]

(b) Water is running at a steady rate of $36\pi \text{ cm}^3 \text{ s}^{-1}$ into a right inverted circular cone with a semi-vertical angle of 45° .

(i) Find the rate of increasing in water depth when the water level is 3 cm.

[4 marks]

(ii) Find the time taken when the depth of the water is 18 cm.

[3 marks]

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- 9 (a) Tentukan nilai bagi R dan α , dengan $R > 0$ dan $0^\circ < \alpha < 90^\circ$ supaya

$$3 \sin \theta - 4 \cos \theta = R \sin(\theta - \alpha).$$

[4 markah]

- (b) Seterusnya, selesaikan persamaan $3 \sin \theta - 4 \cos \theta = 2$ bagi $0^\circ \leq \theta \leq 360^\circ$.

[4 markah]

- (c) Berdasarkan jawapan daripada bahagian (b), cari nilai θ bagi $0^\circ \leq \theta \leq 360^\circ$

supaya $f(\theta) = \frac{1}{3 \sin \theta - 4 \cos \theta + 15}$ adalah minima.

Seterusnya, cari nilai minima bagi f .

[3 markah]

- 10 (a) Cari nilai k jika kecerunan bagi lengkungan $x^3 + kx^2y - 2y^2 = 0$ pada titik $(-1, 1)$ adalah -3 .

[5 markah]

(b) Diberi $y = \frac{\sin x}{1 + \cos x}$.

- (i) Cari $\frac{dy}{dx}$ dan $\frac{d^2y}{dx^2}$ dalam sebutan x .

[5 markah]

(ii) Seterusnya, tunjukkan $\frac{d^3y}{dx^3} - y \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$.

[5 markah]

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KERTAS SOALAN TAMAT

- 9 (a) Determine the values of R and α , where $R > 0$ and $0^\circ < \alpha < 90^\circ$ so that

$$3 \sin \theta - 4 \cos \theta = R \sin(\theta - \alpha).$$

[4 marks]

- (b) Hence, solve the equation $3 \sin \theta - 4 \cos \theta = 2$ for $0^\circ \leq \theta \leq 360^\circ$.

[4 marks]

- (c) From the answer obtained in part (b), find the value of θ for $0^\circ \leq \theta \leq 360^\circ$

so that $f(\theta) = \frac{1}{3 \sin \theta - 4 \cos \theta + 15}$ is minimum.

Hence, find the minimum value of f .

[3 marks]

- 10 (a) Find the value of k if the slope of the curve $x^3 + kx^2y - 2y^2 = 0$ at the point $(-1, 1)$ is -3 .

[5 marks]

(b) Given $y = \frac{\sin x}{1 + \cos x}$.

- (i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x .

[5 marks]

- (ii) Hence, show that $\frac{d^3y}{dx^3} - y \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$.

[5 markah]

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END OF QUESTION PAPER