KEMENTERIAN
PENDIDIKAN
MALAYSIA

QS 015/2

Matriculation Programme Examination

Semester 1

Session 2012/2013

Chow Choon Wooi

1. Given that $f(x) = \begin{cases} 1+e^x, & x<1 \\ 1, & x=1 \\ 2-x, & x>1 \end{cases}$

   Find $\lim\limits_{x\to 1^-} f(x)$ and $\lim\limits_{x\to 1^+} f(x)$. Does the $\lim\limits_{x\to 1} f(x)$ exist? State your reason.

2. Prove that $1+\tan 2\theta \tan\theta = \sec 2\theta$.

3. Find the following limits:

   (a) $\lim\limits_{x\to\infty} \dfrac{2x^2+x-4}{1-x^2}$

   (b) $\lim\limits_{x\to 2} \dfrac{3-\sqrt{x+7}}{x^2-4}$

4. Express $\frac{2x^3-7x^2+17x-19}{2x^2-7x+6}$ in the form of partial fractions.

5. (a) Given that $f(x) = \begin{cases} \frac{|x^2-x-2|}{x^2-2x}, & x \neq 0,2 \\ 0, & x=2 \end{cases}$

   Find the $\lim\limits_{x\to 2} f(x)$. Is $f(x)$ continuous at $x = 2$?

   (b) A function $f(x)$ is defined by $f(x) = \begin{cases} ax+6 & , & x<4 \\ x^2+2 & , & 4\le x<6 \\ 2-\beta x & , & x\ge 6 \end{cases}$

   Determine the values of the constants α and β if $f(x)$ is continuous.

6. The polynomial $P(x) = 2x^3+ax^2+bx-24$ has a factor $(x - 2)$ and a remainder 15 when divided by $(x + 3)$.

   (a) Find the values of a and b.

   (b) Factorise $P(x)$ completely and find all zeroes of $P(x)$.

7. Given $f(\theta) = 3\sin\theta - 2\cos\theta$.

   (a) Express $f(\theta)$ in the form of $R\,sin(\theta-\alpha)$, where $R > 0,\ 0\le\alpha\le\frac{\pi}{2}$. Hence, find the maximum and minimum values of $f(\theta)$.

   (b) Solve $f(\theta) = \sqrt{\frac{3}{2}}\ for\ 0° \le \theta \le 360°$.

8. (a) Given that $y = \frac{1}{\sqrt{2x+1}}$.

   i. By using the first principle of derivative, find $\frac{dy}{dx}$.

   ii. Find $\frac{d^2y}{dx^2}$.

   (b) Find $\frac{dy}{dx}$ of the following:

   i. $y = e^{2x} \tan x$

   ii. $y = x^{\sec x}$

9. (a) A conical tank is of height 12m and surface diameter 8m. Water is pumped into the tank at the rate of 50 m^3/min. How fast is the water level increasing when the depth of the water is 6m?

   (b) A cylindrical container of radius r and height h has a constant volume V. The cost of the materials for the surface of both of its ends is twice the cost of its sides. State h in terms of r and V. Hence, find h and r in terms of V such that the cost is minimum.

10. (a) Given $3y^2 - xy + x^2 = 3$. By using implicit differentiation,

   i. Find the values of $\frac{dy}{dx}$ at $x = 1$.

   ii. Show that $(6y - x)\frac{d^2y}{dx^2} + 6\left(\frac{dy}{dx}\right)^2 - 2\frac{dy}{dx} + 2 = 0$.

   (b) Consider the parametric equations

   $x = 3t - \frac{2}{t}$, $y = 3t + \frac{2}{t}$ where $t \neq 0$.

   i. Show that $\frac{dy}{dx} = 1 - \frac{4}{3t^2+2}$.

   ii. Find $\frac{d^2y}{dx^2}$ when $t = 1$.

END OF QUESTION PAPER

1. Given that $f(x)=\begin{cases}1+e^x, & x<1\\ 1, & x=1\\ 2-x, & x>1\end{cases}$

Find $\lim\limits_{x\to 1^-} f(x)$ and $\lim\limits_{x\to 1^+} f(x)$. Does the $\lim\limits_{x\to 1} f(x)$ exist? State your reason.

SOLUTION

$$f(x)=\begin{cases}1+e^x, & x<1\\ 1, & x=1\\ 2-x, & x>1\end{cases}$$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(1+e^x)$$

$$=(1+e^1)$$

$$=1+e$$

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(2-x)$$

$$=2-1$$

$$=1$$

Since $\lim\limits_{x\to 1^-} f(x)\neq\lim\limits_{x\to 1^+} f(x)$. Therefore $\lim\limits_{x\to 1} f(x)$ does not exist.

2. Prove that $1 + \tan 2\theta \tan\theta = \sec 2\theta$.

SOLUTION

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos(2\theta - \theta) = \cos 2\theta \cos\theta + \sin 2\theta \sin\theta$$

$$1 + \tan 2\theta \tan\theta = 1 + \left(\frac{\sin 2\theta}{\cos 2\,\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$= 1 + \left(\frac{\sin 2\theta \sin\theta}{\cos 2\,\theta \cos\theta}\right)$$

$$= \frac{\cos 2\,\theta \cos\theta + \sin 2\theta \sin\theta}{\cos 2\,\theta \cos\theta}$$

$$= \frac{\cos(2\theta - \theta)}{\cos 2\,\theta \cos\theta}$$

$$= \frac{\cos(\theta)}{\cos 2\,\theta \cos\theta}$$

$$= \frac{1}{\cos 2\,\theta}$$

$$= \sec 2\theta.$$

3. Find the following limits:

(a) $\lim_{x\to\infty}\frac{2x^2+x-4}{1-x^2}$

(b) $\lim_{x\to 2}\frac{3-\sqrt{x+7}}{x^2-4}$

SOLUTION

3(a)

$$\lim_{x\to\infty}\frac{2x^2+x-4}{1-x^2}=\lim_{x\to\infty}\frac{\frac{2x^2+x-4}{x^2}}{\frac{1-x^2}{x^2}}$$

$$=\lim_{x\to\infty}\frac{2+\frac{1}{x}-\frac{4}{x^2}}{\frac{1}{x^2}-1}$$

$$=\frac{2+0-0}{0-1}$$

$$=-2$$

3(b)

$$\lim_{x\to 2}\frac{3-\sqrt{x+7}}{x^2-4}=\lim_{x\to 2}\frac{3-\sqrt{x+7}}{x^2-4}\cdot\frac{3+\sqrt{x+7}}{3+\sqrt{x+7}}$$

$$=\lim_{x\to 2}\frac{9-(x+7)}{(x+2)(x-2)(3+\sqrt{x+7})}$$

$$=\lim_{x\to 2}\frac{2-x}{(x+2)(x-2)(3+\sqrt{x+7})}$$

$$=\lim_{x\to 2}\frac{-(x-2)}{(x+2)(x-2)(3+\sqrt{x+7})}$$

$$=\lim_{x\to 2}\frac{-1}{(x+2)(3+\sqrt{x+7})}$$

$$= \frac{-1}{(2+2)(3+\sqrt{2+7})}$$

$$= \frac{-1}{(4)(3+\sqrt{9})}$$

$$= \frac{-1}{24}$$

4. Express $\frac{2x^3-7x^2+17x-19}{2x^3-7x+6}$ in the form of partial fractions.

SOLUTION

$\frac{2x^3-7x^2+17x-19}{2x^2-7x+6}$ ➔ Improper fraction

$$\begin{array}{r} x \\ 2x^2-7x+6\overline{)2x^3-7x^2+17x-19} \\ \underline{2x^3-7x^2+6x} \\ 11x-19 \end{array}$$

$$\frac{2x^3-7x^2+17x-19}{2x^2-7x+6} = x + \frac{11x-19}{2x^2-7x+6}$$

$$\frac{2x^3-7x^2+17x-19}{2x^2-7x+6} = x + \frac{11x-19}{(2x-3)(x-2)}$$

$$\frac{11x-19}{(2x-3)(x-2)} = \frac{A}{(2x-3)} + \frac{B}{(x-2)}$$

$$\frac{11x-19}{(2x-3)(x-2)} = \frac{A(x-2)+B(2x-3)}{(2x-3)(x-2)}$$

$$11x-19 = A(x-2)+B(2x-3)$$

$When\ x = 2$

$$11(2)-19 = A[(2)-2]+B[(2(2)-3)]$$
$$3 = A[0]+B[(1)]$$
$$B = 3$$

$When\ x = \frac{3}{2}$

$$11\left(\frac{3}{2}\right)-19 = A\left[\left(\frac{3}{2}\right)-2\right]+B\left[(2\left(\frac{3}{2}\right)-3)\right]$$
$$-\frac{5}{2} = A\left[-\frac{1}{2}\right]+B[(0)]$$
$$A = 5$$

$$\frac{11x-19}{(2x-3)(x-2)} = \frac{5}{(2x-3)} + \frac{3}{(x-2)}$$

$$\frac{2x^3-7x^2+17x-19}{2x^2-7x+6} = x + \frac{11x-19}{(2x-3)(x-2)}$$

$$\frac{2x^3-7x^2+17x-19}{2x^2-7x+6} = x + \frac{5}{(2x-3)} + \frac{3}{(x-2)}$$

5. (a) Given that $f(x) = \begin{cases} \frac{|x^2-x-2|}{x^2-2x}, & x \neq 0, 2 \\ 0, & x = 2 \end{cases}$

Find the $\lim_{x\to 2} f(x)$. Is $f(x)$ continuous at $x = 2$?

(b) A function $f(x)$ is defined by $f(x) = \begin{cases} ax + 6 & , \quad x < 4 \\ x^2 + 2 & , \quad 4 \leq x < 6 \\ 2 - \beta x & , \quad x \geq 6 \end{cases}$

Determine the values of the constants α and β if $f(x)$ is continuous.

SOLUTION

5(a)

$$f(x) = \begin{cases} \frac{|x^2-x-2|}{x^2-2x}, & x \neq 0, 2 \\ 0, & x = 2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{|x^2-x-2|}{x^2-2x}$$

$$= \lim_{x\to 2^-} \frac{|(x+1)(x-2)|}{x(x-2)}$$

$$= \lim_{x\to 2^-} \frac{(x+1)[-(x-2)]}{x(x-2)}$$

$$= \lim_{x\to 2^-} \frac{-(x+1)}{x}$$

$$= \frac{-(2+1)}{2}$$

$$= -\frac{3}{2}$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{|x^2-x-2|}{x^2-2x}$$

$$= \lim_{x\to 2^+} \frac{|(x+1)(x-2)|}{x(x-2)}$$

$$= \lim_{x \to 2^+} \frac{(x+1)(x-2)}{x(x-2)}$$

$$= \lim_{x \to 2^+} \frac{x+1}{x}$$

$$= \frac{2+1}{2}$$

$$= \frac{3}{2}$$

$$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$$

$\lim_{x \to 2} f(x)$ does not exist

$f(x)$ discontinuous at $x = 2$

5(b)

$$f(x) = \begin{cases} \alpha x + 6 & , \quad x < 4 \\ x^2 + 2 & , \quad 4 \leq x < 6 \\ 2 - \beta x & , \quad x \geq 6 \end{cases}$$

f is continuous at $x = 4$

$\lim_{x \to 4} f(x)$ exists.

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$$

$$\lim_{x \to 4^-} \alpha x + 6 = \lim_{x \to 4^+} x^2 + 2$$

$$\alpha(4) + 6 = (4)^2 + 2$$

$$4\alpha = 12$$

$$\alpha = 3$$

f is continuous at $x = 6$

$\lim_{x \to 6} f(x)$ exists.

$$\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$$

$$\lim_{x \to 6^-} x^2 + 2 = \lim_{x \to 6^+} 2 - \beta x$$

$$(6)^2 + 2 = 2 - \beta(6)$$

$$38 = 2 - 6\beta$$

$$6\beta = -36$$

$$\beta = -6$$

$$\therefore \alpha = 3, \qquad \beta = -6$$

6. The polynomial $P(x) = 2x^3 + ax^2 + bx - 24$ has a factor $(x - 2)$ and a remainder 15 when divided by $(x + 3)$.

   a. Find the values of a and b.

   b. Factorise $P(x)$ completely and find all zeroes of $P(x)$.

SOLUTION

6(a)

$P(2) = 0$

$P(-3) = 15$

$P(x) = 2x^3 + ax^2 + bx - 24$

$P(2) = 2(2)^3 + a(2)^2 + b(2) - 24 = 0$

$16 + 4a + 2b - 24 = 0$

$4a + 2b = 8$ ....(1)

$P(-3) = 2(-3)^3 + a(-3)^2 + b(-3) - 24 = 15$

$-54 + 9a - 3b - 24 = 15$

$9a - 3b = 93$

$3a - b = 31$.................(2)

$(2)\ X\ 2$

$6a - 2b = 62$.................(3)

(1)+(2)

$10a = 70$

$a = 7$

$b = -10$

6(b)

$$\begin{aligned} P(x) &= 2x^3+7x^2-10x-24 \\ &= (x-2)Q(x) \end{aligned}$$

$$\begin{array}{r r l}
 & & 2x^2+11x+12 \\ \cline{3-3}
x-2 & \big) & 2x^3+7x^2-10x-24 \\
 & & \underline{2x^3-4x^2} \\
 & & 11x^2-10x-24 \\
 & & \underline{11x^2-22x} \\
 & & 12x-24 \\
 & & \underline{12x-24} \\
 & & 0
\end{array}$$

$$\begin{aligned} P(x) &= (x-2)(2x^2+11x+12) \\ &= (x-2)(2x+3)(x+4) \end{aligned}$$

when $P(x)=0$

$$(x-2)(2x+3)(x+4)=0$$

$$x=2\,,\quad x=-\frac{3}{2}\,,\quad x=-4$$

The zeroes are $2\,,\ -\frac{3}{2}$ and -4

7. Given $f(\theta) = 3\sin\theta - 2\cos\theta$.

a. Express $f(\theta)$ in the form of $R\,sin(\theta - \alpha)$, where $R > 0,\ 0 \le \alpha \le \frac{\pi}{2}$. Hence, find the maximum and minimum values of $f(\theta)$.

b. Solve $f(\theta) = \sqrt{\frac{3}{2}}\ for\ 0° \le \theta \le 360°$.

SOLUTION

a) $3\sin\theta - 2\cos\theta = R\sin(\theta-\alpha)$

$$= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$$

$$R\cos\alpha = 3 \qquad (1)$$

$$R\sin\alpha = 2 \qquad (2)$$

$(1)^2 + (2)^2:$

$$R^2\cos^2\alpha + R^2\sin^2\alpha = 3^2 + 2^2$$
$$R^2(1) = 13$$
$$R = \sqrt{13}$$

$(2) \div (1):$

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{2}{3}$$
$$\tan\alpha = \frac{2}{3}$$
$$\alpha = 0.588$$

$$3\sin\theta - 2\cos\theta = \sqrt{13}\sin(\theta - 0.588)$$

$$-1 \le \sin(\theta - 0.588) \le 1$$

$$-\sqrt{13} \le \sqrt{13}\sin(\theta - 0.588) \le \sqrt{13}$$

$$-\sqrt{13} \le 3\sin\theta - 2\cos\theta \le \sqrt{13}$$

$$-\sqrt{13} \le f(\theta) \le \sqrt{13}$$

Minimum value of $f(\theta) = -\sqrt{13}$

Maximum value of $f(\theta) = \sqrt{13}$

b) $f(\theta) = \sqrt{\dfrac{13}{2}}$

$$\sqrt{13}\sin(\theta - 0.588) = \sqrt{\frac{13}{2}}$$

$$\sin(\theta - 0.588) = \frac{1}{\sqrt{2}}$$

$$\theta - 0.588 = 0.785\,,\quad \pi - 0.785$$

$$\theta = 1.373\,,\ 2.945$$

$$\theta = 78.7^0\,,\ 168.7^0$$

8. (a) Given that $y = \frac{1}{\sqrt{2x+1}}$.

i. By using the first principle of derivative, find $\frac{dy}{dx}$.

ii. Find $\frac{d^2y}{dx^2}$.

(b) Find $\frac{dy}{dx}$ of the following:

iii. $y = e^{2x} \tan x$

iv. $y = x^{\sec x}$

SOLUTION

(a) $y = \dfrac{1}{\sqrt{2x+1}}$

i) Let $f(x) = \dfrac{1}{\sqrt{2x+1}}$

$$f(x+h) = \frac{1}{\sqrt{2(x+h)+1}}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\dfrac{1}{\sqrt{2(x+h)+1}} - \dfrac{1}{\sqrt{2x+1}}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{\sqrt{2x+1} - \sqrt{2(x+h)+1}}{\sqrt{2(x+h)+1}\sqrt{2x+1}}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2x+1} - \sqrt{2x+2h+1}}{h\sqrt{2(x+h)+1}\sqrt{2x+1}} \frac{\sqrt{2x+1} + \sqrt{2x+2h+1}}{\sqrt{2x+1} + \sqrt{2x+2h+1}}$$

$$= \lim_{h \to 0} \frac{(2x+1) - (2x+2h+1)}{h\sqrt{2(x+h)+1}\sqrt{2x+1}(\sqrt{2x+1} + \sqrt{2x+2h+1})}$$

$$= \lim_{h \to 0} \frac{-2h}{h\sqrt{2(x+h)+1}\sqrt{2x+1}(\sqrt{2x+1} + \sqrt{2x+2h+1})}$$

$$= \lim_{h \to 0} \frac{-2}{\sqrt{2(x+h)+1}\sqrt{2x+1}(\sqrt{2x+1} + \sqrt{2x+2h+1})}$$

$$= \frac{-2}{\sqrt{2x+1}\sqrt{2x+1}(\sqrt{2x+1}+\sqrt{2x+1})}$$

$$= \frac{-2}{(2x+1)(2\sqrt{2x+1})}$$

$$= -\frac{1}{(2x+1)^{\frac{3}{2}}}$$

ii) $\dfrac{dy}{dx} = -(2x+1)^{-\frac{3}{2}}$

$$\frac{d^2y}{dx^2} = \frac{3}{2}(2x+1)^{-\frac{5}{2}}(2)$$

$$= \frac{3}{(2x+1)^{\frac{5}{2}}}$$

b i) $y = e^{2x}\tan x$

$$\frac{dy}{dx} = e^{2x}(\sec^2 x) + \tan x(2e^{2x})$$

$$= e^{2x}(\sec^2 x + 2\tan x)$$

ii) $y = x^{\sec x}$

$$\ln y = \ln x^{\sec x}$$

$$\ln y = \sec x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \sec x\left(\frac{1}{x}\right) + (\ln x)(\sec x \tan x)$$

$$\frac{dy}{dx} = y\left[\frac{\sec x + x(\ln x)\sec x \tan x}{x}\right]$$

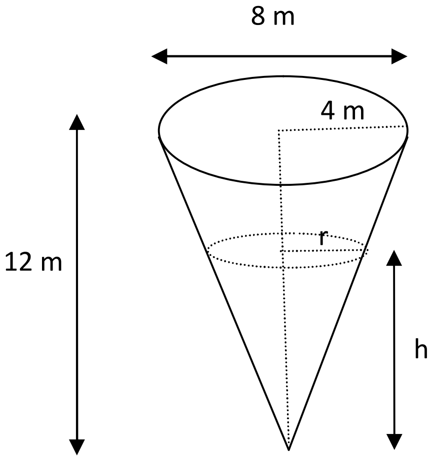
$$= x^{\sec x}\sec x\left[\frac{1 + x(\ln x)\tan x}{x}\right]$$

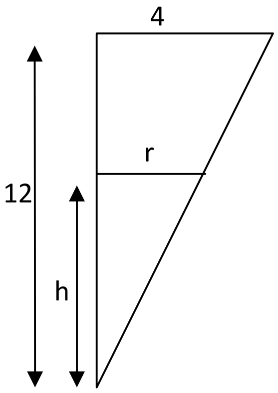
9. (a) A conical tank is of height 12m and surface diameter 8m. Water is pumped into the tank at the rate of 50 m^3/min. How fast is the water level increasing when the depth of the water is 6m?

(b) A cylindrical container of radius r and height h has a constant volume V. The cost of the materials for the surface of both of its ends is twice the cost of its sides. State h in terms of r and V. Hence, find h and r in terms of V such that the cost is minimum.

SOLUTION

9(a)





[redacted]

$$\frac{dV}{dt} = 50\ m^3/min \qquad \left[\frac{dh}{dt} = ? \quad when\ h = 6\right]$$

$$\frac{dh}{dt} = \frac{dh}{dv} \cdot \frac{dv}{dt}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$$

$$V = \frac{1}{3}\pi\frac{h^3}{9}$$

$$V = \frac{1}{27}\pi h^3$$

$$\frac{dV}{dh} = \frac{1}{9}\pi h^2 = \frac{\pi h^2}{9}$$

$$\frac{dh}{dV} = \frac{9}{\pi h^2}$$

$$\frac{dh}{dt} = \frac{dh}{dv} \cdot \frac{dv}{dt}$$

$$\frac{dh}{dt} = \left(\frac{9}{\pi h^2}\right) . (50)$$

$$\frac{dh}{dt} = \frac{450}{\pi h^2}$$

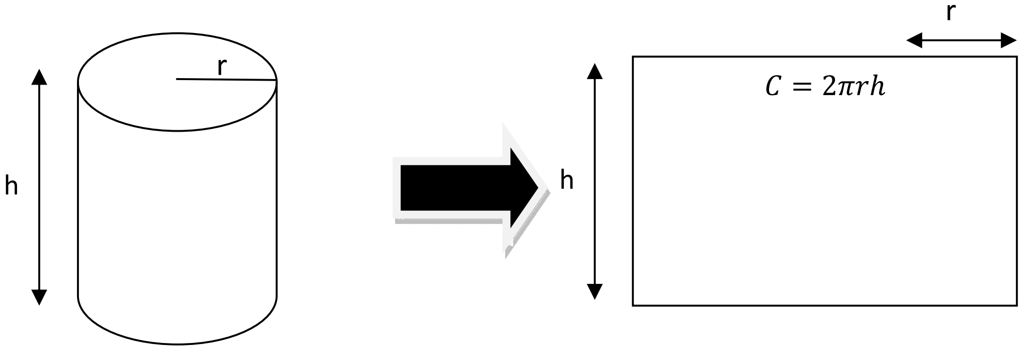
$When\, h \ = \ 6$

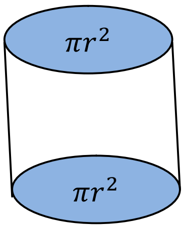
$$\frac{dh}{dt} = \frac{450}{\pi (6)^2}$$

$$\frac{dh}{dt} = \frac{450}{36\pi}$$

$$\frac{dh}{dt} = \frac{25}{2\pi} \ m/min$$

9(b)





$$V = \pi r^2 h$$

$$\therefore h = \frac{V}{\pi r^2}$$

The cost,

$$C = 2\pi r^2(2) + 2\pi rh(1)$$

$$= 4\pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right)$$

$$C = 4\pi r^2 + \frac{2V}{r}$$

$$\frac{dC}{dr} = 8\pi r - \frac{2V}{r^2}$$

when $\frac{dC}{dr} = 0$

$$8\pi r - \frac{2V}{r^2} = 0$$

$$8\pi r = \frac{2V}{r^2}$$

$$r^3 = \frac{V}{4\pi}$$

$$r = \left(\frac{V}{4\pi}\right)^{\frac{1}{3}}$$

$$\frac{d^2C}{dr^2} = 8\pi + \frac{4V}{r^3}$$

when $r = \left(\frac{V}{4\pi}\right)^{\frac{1}{3}}$,

$$\frac{d^2C}{dr^2} = 8\pi + \frac{4V}{\frac{V}{4\pi}} = 24\pi > 0 \text{ (min)}$$

$$\therefore r = \left(\frac{V}{4\pi}\right)^{\frac{1}{3}}$$

$$\therefore h = \frac{V}{\pi r^2} = \frac{V}{\pi\left(\frac{V}{4\pi}\right)^{\frac{2}{3}}} = \left(\frac{16V}{\pi}\right)^{\frac{1}{3}}$$

10 (a) Given $3y^2 - xy + x^2 = 3$. By using implicit differentiation,

i. Find the values of $\frac{dy}{dx}$ at $x = 1$.

ii. Show that $(6y - x)\frac{d^2y}{dx^2} + 6\left(\frac{dy}{dx}\right)^2 - 2\frac{dy}{dx} + 2 = 0$.

(b) Consider the parametric equations

$x = 3t - \frac{2}{t}$, $y = 3t + \frac{2}{t}$ where $t \neq 0$.

iii. Show that $\frac{dy}{dx} = 1 - \frac{4}{3t^2+2}$.

iv. Find $\frac{d^2y}{dx^2}$ when $t = 1$.

SOLUTION

10[a(i))]

$$3y^2 - xy + x^2 = 3$$

$$6y\frac{dy}{dx} - \left[x\frac{dy}{dx} + y\right] + 2x = 0$$

$$6y\frac{dy}{dx} - x\frac{dy}{dx} - y + 2x = 0$$

$$(6y - x)\frac{dy}{dx} = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{6y - x}$$

$When\ x\ =\ 1$

$$3y^2 - (1)y + (1)^2 = 3$$

$$3y^2 - y + 1 = 3$$

$$3y^2 - y - 2 = 0$$

$$3y^2 - y - 2 = 0$$

$$(3y + 2)(y - 1) = 0$$

$$y = -\frac{2}{3}\ or\ y = 1$$

$$6y\frac{dy}{dx}-x\left(\frac{dy}{dx}\right)+y(-1)+2x=0$$

$$\frac{dy}{dx}(6y-x)=y-2x$$

$$\frac{dy}{dx}=\frac{y-2x}{6y-x}$$

For $x=1,\ y=-\frac{2}{3}$:

$$\frac{dy}{dx}=\frac{\left(-\frac{2}{3}\right)-2(1)}{6\left(-\frac{2}{3}\right)-1}=\frac{8}{15}$$

For $x=1,\ y=1$:

$$\frac{dy}{dx}=\frac{1-2(1)}{6(1)-1}=-\frac{1}{5}$$

10[a(ii))]

$$\frac{dy}{dx}(6y-x)=y-2x$$

$$\frac{dy}{dx}\left(6\frac{dy}{dx}-1\right)+(6y-x)\left(\frac{d^2y}{dx^2}\right)=\frac{dy}{dx}-2$$

$$6\left(\frac{dy}{dx}\right)^2-\frac{dy}{dx}+(6y-x)\left(\frac{d^2y}{dx^2}\right)-\frac{dy}{dx}+2=0$$

$$(6y-x)\frac{d^2y}{dx^2}+6\left(\frac{dy}{dx}\right)^2-2\frac{dy}{dx}+2=0$$

10(b)

$$x=3t-\frac{2}{t}\ ,\quad y=3t+\frac{2}{t}$$

i) $$\frac{dx}{dt}=3+\frac{2}{t^2}=\frac{3t^2+2}{t^2}$$

$$\frac{dy}{dt}=3-\frac{2}{t^2}=\frac{3t^2-2}{t^2}$$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{\dfrac{3t^2 - 2}{t^2}}{\dfrac{3t^2 + 2}{t^2}}$$

$$= \frac{3t^2 - 2}{3t^2 + 2}$$

$$\begin{array}{r|l} & \quad 1 \\ \cline{2-2} 3t^2 + 2 & 3t^2 - 2 \\ & \underline{3t^2 + 2} \\ & \quad -4 \end{array}$$

$$\frac{dy}{dx} = 1 - \frac{4}{3t^2 + 2}$$

ii) $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dt}\left(1 - \dfrac{4}{3t^2 + 2}\right)$

$$= \frac{d}{dt}[1 - 4(3t^2 + 2)^{-1}]$$

$$= 4(3t^2 + 2)^{-2}(6t)$$

$$= \frac{24t}{(3t^2 + 2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)}{\dfrac{dx}{dt}} = \frac{\dfrac{24t}{(3t^2 + 2)^2}}{\dfrac{3t^2 + 2}{t^2}} = \frac{24t^3}{(3t^2 + 2)^3}$$

when $t = 1$,

$$\frac{d^2y}{dx^2} = \frac{24(1)^3}{(3(1)^2 + 2)^3} = \frac{24}{125}$$