QS015/2
Mathematics
Paper 2
Semester I
Session 2012/2013
2 hours

QS015/2
Matematik
Kertas 2
Semester I
Sesi 2012/2013
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **17** halaman bercetak.
This question paper consists of **17** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin\left(A \pm B\right) = \sin A \cos B \pm \cos A \sin B$$

$$\cos\left(A \pm B\right) = \cos A \cos B \mp \sin A \sin B$$

$$\tan\left(A \pm B\right) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

LIST OF MATHEMATICAL FORMULAE

Limit

$$\lim_{h\to 0}\frac{\sin h}{h}=1$$

$$\lim_{h\to 0}\frac{1-\cos h}{h}=0$$

Differentiation

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

If $y=g(t)$ and $x=f(t)$, then $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Sphere $V=\frac{4}{3}\pi r^3$ $S=4\pi r^2$

Right circular cone $V=\frac{1}{3}\pi r^2 h$ $S=\pi rs$

Right circular cylinder $V=\pi r^2 h$ $S=2\pi rh$

1 Given that $f(x)=\begin{cases}1+e^{x}, & x<1\\ 1, & x=1\\ 2-x, & x>1.\end{cases}$

Find $\lim\limits_{x\to 1^{-}} f(x)$ and $\lim\limits_{x\to 1^{+}} f(x)$. Does the $\lim\limits_{x\to 1} f(x)$ exist? State your reason.

[5 *marks*]

2 Prove that $1+\tan 2\theta \tan\theta = \sec 2\theta$.

[6 *marks*]

3 Find the following limits:

(a) $\lim\limits_{x\to\infty} \dfrac{2x^{2}+x-4}{1-x^{2}}$.

[3 *marks*]

(b) $\lim\limits_{x\to 2} \dfrac{3-\sqrt{x+7}}{x^{2}-4}$.

[4 *marks*]

4 Express $\dfrac{2x^{3}-7x^{2}+17x-19}{2x^{2}-7x+6}$ in the form of partial fractions.

[7 *marks*]

5 (a) Given that $f(x)=\begin{cases}\dfrac{|x^2-x-2|}{x^2-2x}, & x\neq 0,2\\ 0, & x=2.\end{cases}$

Find the $\lim_{x\to 2} f(x)$. Is $f(x)$ continuous at $x=2$?

[6 *marks*]

(b) A function $f(x)$ is defined by $f(x)=\begin{cases}\alpha x+6, & x<4\\ x^2+2, & 4\leq x<6\\ 2-\beta x, & x\geq 6.\end{cases}$

Determine the values of the constants α and β if $f(x)$ is continuous.

[5 *marks*]

6 The polynomial $P(x)=2x^3+ax^2+bx-24$ has a factor $(x-2)$ and a remainder 15 when divided by $(x+3)$.

(a) Find the values of a and b.

[6 *marks*]

(b) Factorise $P(x)$ completely and find all zeroes of $P(x)$.

[6 *marks*]

7 Given $f(\theta) = 3\sin\theta - 2\cos\theta$.

(a) Express $f(\theta)$ in the form of $R\sin(\theta - \alpha)$, where $R > 0$, $0 \le \alpha \le \frac{\pi}{2}$.

Hence, find the maximum and minimum values of $f(\theta)$.

[8 *marks*]

(b) Solve $f(\theta) = \sqrt{\frac{13}{2}}$ for $0^0 \le \theta \le 360^0$.

[4 *marks*]

8 (a) Given that $y = \frac{1}{\sqrt{2x+1}}$.

(i) By using the first principle of derivative, find $\frac{dy}{dx}$.

[4 *marks*]

(ii) Find $\frac{d^2y}{dx^2}$.

[2 *marks*]

(b) Find $\frac{dy}{dx}$ of the following:

(i) $y = e^{2x}\tan x$.

[2 *marks*]

(ii) $y = x^{\sec x}$.

[4 *marks*]

9 (a) A conical tank is of height 12 m and surface diameter 8 m. Water is pumped into the tank at the rate of 50 m^3/min. How fast is the water level increasing when the depth of the water is 6 m?

[6 *marks*]

(b) A cylindrical container of radius r and height h has a constant volume V. The cost of the materials for the surface of both of its ends is twice the cost of its sides. State h in terms of r and V. Hence, find h and r in terms of V such that the cost is minimum.

[7 *marks*]

10 (a) Given $3y^2 - xy + x^2 = 3$. By using implicit differentiation,

(i) find the value of $\frac{dy}{dx}$ at $x = 1$.

[6 *marks*]

(ii) show that $(6y - x)\frac{d^2y}{dx^2} + 6\left(\frac{dy}{dx}\right)^2 - 2\frac{dy}{dx} + 2 = 0$.

[2 *marks*]

(b) Consider the parametric equations

$$x = 3t - \frac{2}{t},\ y = 3t + \frac{2}{t} \text{ where } t \neq 0.$$

(i) Show that $\frac{dy}{dx} = 1 - \frac{4}{3t^2 + 2}$.

[3 *marks*]

(ii) Find $\frac{d^2y}{dx^2}$ when $t = 1$.

[4 *marks*]

END OF QUESTION PAPER