



**QS 015/1**  
**Matriculation Programme**  
**Examination**  
**Semester 1**  
**Session 2012/2013**

1. Find the value of  $x$  which satisfies the equation

$$\log_2(5 - x) - \log_2(x - 2) = 3 - \log_2(1 + x)$$

2. Determine the solution set of the inequality

$$\frac{1}{2x - 1} < \frac{1}{x + 2}$$

3. Given  $k + 2, k - 4, k - 7$  are the first three terms of a geometric series. Determine the value of  $k$ . Hence, find the sum to infinity of the series.

4. Given a complex number  $z = 1 - \sqrt{3}i$ . Determine the value of  $k$  if  $\overline{z^2} = k\frac{1}{z}$ .

5. (a) Matrix  $M$  is given as  $\begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$ . Show that  $M^2 = 7M - 8I$ , where  $I$  is the  $2 \times 2$  identity matrix. Deduce that  $M^{-1} = \frac{7}{8}I - \frac{1}{8}M$ .

- (b) Given matrix  $A = \begin{bmatrix} p + 1 & -1 & 1 \\ 3 & 2 & 4 \\ -1 & 0 & p + 2 \end{bmatrix}$  and  $|A| = 27$ . Find the value of  $p$ , where  $p$  is an integer.

6. The functions  $f$  and  $g$  are defined as  $f(x) = \frac{3x+4}{x-2}$ ,  $x \neq 2$  and  $g(x) = 3 - x$ .

(a) Find  $f^{-1}(x)$  and  $g^{-1}(x)$ .

(b) Evaluate  $(f \circ g^{-1})(3)$ .

(c) If  $(g \circ f^{-1})(k) = \frac{2}{3}$ , find the value of  $k$ .

7. (a) Solve  $|x^2 - x - 3| = 3$ .

(b) Find the solution set of the inequality  $\frac{2x^2+9x-4}{x+2} < 4$ .

8. The first four terms of a binomial expansion  $(1 + ax)^n$  is

$$1 + x - \frac{1}{2}x^2 + px^3 + \dots$$

Find

- (a) the values of  $a$  and  $n$  where  $n \neq 0$ .

- (b) the value of  $p$ . Hence, by substituting  $x = \frac{1}{4}$ , show that  $\sqrt{\frac{3}{2}}$  is approximately equal to  $\frac{157}{128}$ .
9. Given  $f(x) = \ln(2x + 3)$  and  $g(x) = \frac{e^x - 3}{2}$ .
- (a) Show that  $f(x)$  is a one-to-one function algebraically.
- (b) Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ . Hence, state the conclusion about the results.
- (c) Sketch the graphs of  $f(x)$  and  $g(x)$  on the same axes. Hence, state the domain and range of  $f(x)$ .
10. Given
- $$A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix}$$
- (a) Find the determinant of matrix  $A$ .
- (b) Find the minor, cofactor and adjoint of matrix  $A$ .
- (c) Given  $A(\text{adjoint}(A)) = |A|I$  where  $I$  is  $3 \times 3$  identity matrix, show that  $A^{-1} = \frac{1}{|A|} \text{adjoint}(A)$ . Hence, find  $A^{-1}$ .
- (d) By using  $A^{-1}$  in part (c), solve the following simultaneous equations.
- $$\begin{aligned} 2x + 2y + 3z &= 49 \\ x + 5y + 4z &= 74 \\ 3x + y + 4z &= 49 \end{aligned}$$

**END OF QUESTION PAPER**

1. Find the value of  $x$  which satisfies the equation

$$\log_2(5 - x) - \log_2(x - 2) = 3 - \log_2(1 + x)$$

**SOLUTION**

$$\log_2(5 - x) - \log_2(x - 2) = 3 - \log_2(1 + x)$$

$$\log_2 \frac{(5 - x)}{(x - 2)} = 3 - \log_2(1 + x)$$

$$\log_2 \frac{(5 - x)}{(x - 2)} + \log_2(1 + x) = 3$$

$$\log_2 \frac{(5 - x)(1 + x)}{(x - 2)} = 3$$

$$\frac{(5 - x)(1 + x)}{(x - 2)} = 2^3$$

$$\frac{(5 - x)(1 + x)}{(x - 2)} = 8$$

$$(5 - x)(1 + x) = 8(x - 2)$$

$$5 + 5x - x - x^2 = 8x - 16$$

$$5 + 4x - x^2 = 8x - 16$$

$$0 = x^2 + 8x - 4x - 16 - 5$$

$$x^2 + 8x - 4x - 16 - 5 = 0$$

$$x^2 + 4x - 21 = 0$$

$$(x + 7)(x - 3) = 0$$

$$(x + 7) = 0$$

$$x = -7$$

$$\text{or } (x - 3) = 0$$

$$\text{or } x = 3$$

$$\log_a b = c \rightarrow b = a^c$$

**Checking**

$$\log_2(5 - x) - \log_2(x - 2) = 3 - \log_2(1 + x)$$

When  $x = -7$

$$\log_2[5 - (-7)] - \log_2(-7 - 2) = 3 - \log_2(1 - 7)$$

$$x \neq -7$$

When  $x = 3$

$$\log_2(5 - 3) - \log_2(3 - 2) = 3 - \log_2(1 + 3)$$

$$\log_2(2) - \log_2(1) = 3 - \log_2(4)$$

$$1 - 0 = 3 - 2$$

$$1 = 1 \checkmark$$

$$\therefore x = 3$$

2. Determine the solution set of the inequality

$$\frac{1}{2x-1} < \frac{1}{x+2}$$

**SOLUTION**

$$\frac{1}{2x-1} < \frac{1}{x+2}$$

$$\frac{1}{2x-1} - \frac{1}{x+2} < 0$$

$$\frac{1(x+2) - 1(2x-1)}{(2x-1)(x+2)} < 0$$

$$\frac{x+2-2x+1}{(2x-1)(x+2)} < 0$$

$$\frac{-x+3}{(2x-1)(x+2)} < 0$$

$$-x+3=0$$

$$2x-1=0$$

$$x+2=0$$

$$x=3$$

$$x=\frac{1}{2}$$

$$x=-2$$

	$(-\infty, -2)$	$(-2, \frac{1}{2})$	$(\frac{1}{2}, 3)$	$(3, \infty)$
$-x+3$	+	+	+	-
$2x-1$	-	-	+	+
$x+2$	-	+	+	+
$\frac{-x+3}{(2x-1)(x+2)}$	+	-	+	-

$\therefore$  The solution is  $(-2, \frac{1}{2}) \cup (3, \infty)$

3. Given  $k + 2, k - 4, k - 7$  are the first three terms of a geometric series. Determine the value of  $k$ . Hence, find the sum to infinity of the series.

**SOLUTION**

$$k + 2, k - 4, k - 7 \dots$$

$$\frac{k - 4}{k + 2} = \frac{k - 7}{k - 4}$$

$$(k - 4)(k - 4) = (k - 7)(k + 2)$$

$$k^2 - 4k - 4k + 16 = k^2 + 2k - 7k - 14$$

$$k^2 - 8k + 16 = k^2 - 5k - 14$$

$$16 + 14 = -5k + 8k$$

$$3k = 30$$

$$k = 10$$

When  $k = 10$ , the sequence is

$$10 + 2, 10 - 4, 10 - 7 \dots$$

$$12, 6, 3 \dots$$

$$a = 12, \quad r = \frac{6}{12} = \frac{1}{2}$$

$$S_{\infty} = \frac{a}{1 - r}$$

$$S_{\infty} = \frac{12}{1 - \left(\frac{1}{2}\right)}$$

$$S_{\infty} = \frac{12}{\left(\frac{1}{2}\right)}$$

$$S_{\infty} = 24$$

4. Given a complex number  $z = 1 - \sqrt{3}i$ . Determine the value of  $k$  if  $\overline{z^2} = k \frac{1}{z}$ .

**SOLUTION**

$$z = 1 - \sqrt{3}i \qquad \bar{z} = 1 + \sqrt{3}i$$

$$\overline{z^2} = k \frac{1}{z}$$

$$\overline{(1 - \sqrt{3}i)^2} = k \left( \frac{1}{1 + \sqrt{3}i} \right)$$

$$\overline{(1 - \sqrt{3}i)(1 - \sqrt{3}i)} = \frac{k}{1 + \sqrt{3}i}$$

$$\overline{1 - 2\sqrt{3}i + 3i^2} = \frac{k}{1 + \sqrt{3}i}$$

$$\overline{1 - 2\sqrt{3}i + 3(-1)} = \frac{k}{1 + \sqrt{3}i}$$

$$\overline{1 - 2\sqrt{3}i - 3} = \frac{k}{1 + \sqrt{3}i}$$

$$\overline{-2 - 2\sqrt{3}i} = \frac{k}{1 + \sqrt{3}i}$$

$$-2 + 2\sqrt{3}i = \frac{k}{1 + \sqrt{3}i}$$

$$k = (-2 + 2\sqrt{3}i)(1 + \sqrt{3}i)$$

$$k = -2 - 2\sqrt{3}i + 2\sqrt{3}i + 6i^2$$

$$k = -2 + 6(-1)$$

$$k = -8$$

5. (a) Matrix  $M$  is given as  $\begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$ . Show that  $M^2 = 7M - 8I$ , where  $I$  is the  $2 \times 2$  identity matrix. Deduce that  $M^{-1} = \frac{7}{8}I - \frac{1}{8}M$ .

- (b) Given matrix  $A = \begin{bmatrix} p+1 & -1 & 1 \\ 3 & 2 & 4 \\ -1 & 0 & p+2 \end{bmatrix}$  and  $|A| = 27$ . Find the value of  $p$ , where  $p$  is an integer.

### SOLUTION

#### 5(a)

$$M = \begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 9+4 & -3-4 \\ -12-16 & 4+16 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 13 & -7 \\ -28 & 20 \end{bmatrix}$$

$$7M - 8I = 7 \begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix} - 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$7M - 8I = \begin{bmatrix} 21 & -7 \\ -28 & 28 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$7M - 8I = \begin{bmatrix} 13 & -7 \\ -28 & 20 \end{bmatrix}$$

$$\therefore M^2 = 7M - 8I$$

$$M^2 = 7M - 8I$$

$$M^2 - 7M = -8I$$

$$M(M - 7I) = -8I$$

$$M^{-1} = -\frac{1}{8}(M - 7I)$$

$$M^{-1} = -\frac{1}{8}M + \frac{7}{8}I$$

$$M^{-1} = \frac{7}{8}I - \frac{1}{8}M$$



**5(b)**

$$A = \begin{bmatrix} p+1 & -1 & 1 \\ 3 & 2 & 4 \\ -1 & 0 & p+2 \end{bmatrix}$$

$$|A| = 27$$

$$|A| = +(p+1) \begin{vmatrix} 2 & 4 \\ 0 & p+2 \end{vmatrix} - (-1) \begin{vmatrix} 3 & 4 \\ -1 & p+2 \end{vmatrix} + (1) \begin{vmatrix} 3 & 2 \\ -1 & 0 \end{vmatrix}$$

$$|A| = +(p+1)[(2p+4) - (0)] + [(3p+6) - (-4)] + [(0) - (-2)]$$

$$|A| = (p+1)(2p+4) + (3p+6+4) + 2$$

$$|A| = 2p^2 + 4p + 2p + 4 + 3p + 6 + 4 + 2$$

$$|A| = 2p^2 + 9p + 16$$

$$2p^2 + 9p + 16 = 27$$

$$2p^2 + 9p - 11 = 0$$

$$(2p+11)(p-1) = 0$$

$$p = -\frac{11}{2} \quad p = 1$$

Since  $p$  is an integer,  $\therefore p = 1$

6. The functions  $f$  and  $g$  are defined as  $f(x) = \frac{3x+4}{x-2}$ ,  $x \neq 2$  and  $g(x) = 3 - x$ .

- (a) Find  $f^{-1}(x)$  and  $g^{-1}(x)$ .
- (b) Evaluate  $(f \circ g^{-1})(3)$ .
- (c) If  $(g \circ f^{-1})(k) = \frac{2}{3}$ , find the value of  $k$ .

### SOLUTION

#### 6(a)

$f(x) = \frac{3x+4}{x-2}$ $y = \frac{3x+4}{x-2}$ $y(x-2) = 3x+4$ $xy - 2y = 3x+4$ $xy - 3x = 4 + 2y$ $x(y-3) = 4 + 2y$ $x = \frac{4+2y}{y-3}$ $f^{-1}(x) = \frac{4+2x}{x-3}$	$g(x) = 3 - x$ $y = 3 - x$ $x = 3 - y$ $g^{-1}(x) = 3 - x$
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#### 6(b)

$$f(x) = \frac{3x+4}{x-2} \qquad f^{-1}(x) = \frac{4+2x}{x-3}$$

$$g(x) = 3 - x \qquad g^{-1}(x) = 3 - x$$

$$\begin{aligned} (f \circ g^{-1})(x) &= f[g^{-1}(x)] \\ &= f[3 - x] \\ &= \frac{3(3 - x) + 4}{(3 - x) - 2} \end{aligned}$$

$$= \frac{9 - 3x + 4}{3 - x - 2}$$

$$= \frac{13 - 3x}{1 - x}$$

$$(f \circ g^{-1})(3) = \frac{13 - 3(3)}{1 - (3)}$$

$$(f \circ g^{-1})(3) = \frac{13 - 9}{-2}$$

$$(f \circ g^{-1})(3) = -2$$

**6(c)**

$$f(x) = \frac{3x+4}{x-2} \qquad f^{-1}(x) = \frac{4+2x}{x-3}$$

$$g(x) = 3 - x \qquad g^{-1}(x) = 3 - x$$

$$(g \circ f^{-1})(k) = \frac{2}{3}$$

$$(g \circ f^{-1})(k) = g[f^{-1}(k)]$$

$$= g\left[\frac{4+2k}{k-3}\right]$$

$$= 3 - \left(\frac{4+2k}{k-3}\right)$$

$$3 - \left(\frac{4+2k}{k-3}\right) = \frac{2}{3}$$

$$\frac{4+2k}{k-3} = 3 - \frac{2}{3}$$

$$\frac{4+2k}{k-3} = \frac{7}{3}$$

$$3(4+2k) = 7(k-3)$$

$$12 + 6k = 7k - 21$$

$$k = 33$$

7. (a) Solve  $|x^2 - x - 3| = 3$ .
- (b) Find the solution set of the inequality  $\frac{2x^2+9x-4}{x+2} < 4$ .

**SOLUTION****7(a)**

$$|x^2 - x - 3| = 3$$

$$x^2 - x - 3 = 3 \quad \text{or} \quad x^2 - x - 3 = -3$$

$$x^2 - x - 6 = 0 \quad \text{or} \quad x^2 - x = 0$$

$$(x - 3)(x + 2) = 0 \quad \text{or} \quad x(x - 1) = 0$$

$$x = 3 \text{ or } x = -2 \quad \text{or} \quad x = 0 \text{ or } x = 1$$

**7(b)**

$$\frac{2x^2 + 9x - 4}{x + 2} < 4$$

$$\frac{2x^2 + 9x - 4}{x + 2} - 4 < 0$$

$$\frac{(2x^2 + 9x - 4) - 4(x + 2)}{x + 2} < 0$$

$$\frac{2x^2 + 9x - 4 - 4x - 8}{x + 2} < 0$$

$$\frac{2x^2 + 5x - 12}{x + 2} < 0$$

$$\frac{(2x - 3)(x + 4)}{x + 2} < 0$$

$$2x - 3 = 0 \quad x + 4 = 0 \quad x + 2 = 0$$

$$x = \frac{3}{2} \quad x = -4 \quad x = -2$$

	$(-\infty, -4)$	$(-4, -2)$	$(-2, \frac{3}{2})$	$(\frac{3}{2}, \infty)$
$2x - 3$	-	-	-	+
$x + 4$	-	+	+	+
$x + 2$	-	-	+	+
$\frac{(2x - 3)(x + 4)}{x + 2}$	-	+	-	+

$\therefore$  The solution set  $\left\{x: x < -4 \cup -2 < x < \frac{3}{2}\right\}$

8. The first four terms of a binomial expansion  $(1 + ax)^n$  is

$$1 + x - \frac{1}{2}x^2 + px^3 + \dots$$

Find

- (a) the values of a and n where  $n \neq 0$ .
- (b) the value of p Hence, by substituting  $x = \frac{1}{4}$ , show that  $\sqrt{\frac{3}{2}}$  is approximately equal to  $\frac{157}{128}$ .

**SOLUTION**

**8(a)**

Given that

$$(1 + ax)^n = 1 + x - \frac{1}{2}x^2 + px^3 + \dots$$

From binomial expansion

$$(1 + ax)^n = 1 + (n)(ax)^1 + \left(\frac{n(n-1)}{2!}\right)(ax)^2 + \left(\frac{n(n-1)(n-2)}{3!}\right)(ax)^3 + \dots$$

$$(1 + ax)^n = 1 + nax + \frac{n(n-1)}{2}a^2x^2 + \frac{n(n-1)(n-2)}{6}a^3x^3 + \dots$$

$$1 + x - \frac{1}{2}x^2 + px^3 + \dots = 1 + nax + \frac{n(n-1)}{2}a^2x^2 + \frac{n(n-1)(n-2)}{6}a^3x^3 + \dots$$

Coefficient of  $x$ :

$$na = 1 \dots\dots\dots (1)$$

Coefficient of  $x^2$ :

$$\frac{n(n-1)}{2}a^2 = -\frac{1}{2}$$

$$n(n-1)a^2 = -1$$

$$naa(n-1) = -1$$

$$a(n-1) = -1 \dots\dots\dots (2)$$

$na = 1$

$$\frac{(2)}{(1)} \quad \frac{a(n-1)}{na} = \frac{-1}{1}$$

$$\frac{(n-1)}{n} = -1$$

$$n-1 = -n$$

$$2n = 1$$

$$n = \frac{1}{2}$$

From (1)

$$na = 1$$

$$\left(\frac{1}{2}\right)a = 1$$

$$a = 2$$

### 8(b)

Compare coefficient of  $x^3$ :

$$\frac{n(n-1)(n-2)}{6}a^3 = p$$

$$\frac{\left(\frac{1}{2}\right)\left[\left(\frac{1}{2}\right)-1\right]\left[\left(\frac{1}{2}\right)-2\right]}{6}(2)^3 = p$$

$$\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(8)}{6} = p$$

$$\frac{24}{8} = 6p$$

$$6p = 3$$

$$p = \frac{1}{2}$$

$$(1+2x)^{\frac{1}{2}} = 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots$$

When  $x = \frac{1}{4}$

$$\left[1 + 2\left(\frac{1}{4}\right)\right]^{\frac{1}{2}} = 1 + \frac{1}{4} - \frac{1}{2}\left(\frac{1}{4}\right)^2 + \frac{1}{2}\left(\frac{1}{4}\right)^3 + \dots$$

$$\left[1 + \frac{1}{2}\right]^{\frac{1}{2}} = 1 + \frac{1}{4} - \frac{1}{2}\left(\frac{1}{16}\right) + \frac{1}{2}\left(\frac{1}{64}\right) + \dots$$

$$\left[\frac{3}{2}\right]^{\frac{1}{2}} \approx 1 + \frac{1}{4} - \frac{1}{32} + \frac{1}{128}$$

$$\left[\frac{3}{2}\right]^{\frac{1}{2}} \approx \frac{1(128) + 1(32) - 1(4) + 1}{128}$$

$$\left[\frac{3}{2}\right]^{\frac{1}{2}} \approx \frac{157}{128}$$

$$\sqrt{\frac{3}{2}} \approx \frac{157}{128}$$



9. Given  $f(x) = \ln(2x + 3)$  and  $g(x) = \frac{e^x - 3}{2}$ .
- (a) Show that  $f(x)$  is a one-to-one function algebraically.
- (b) Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ . Hence, state the conclusion about the results.
- (c) Sketch the graphs of  $f(x)$  and  $g(x)$  on the same axes. Hence, state the domain and range of  $f(x)$ .

**SOLUTION****9(a)**

$$f(x) = \ln(2x + 3)$$

$$f(x_1) = \ln(2x_1 + 3)$$

$$f(x_2) = \ln(2x_2 + 3)$$

$$\text{Let } f(x_1) = f(x_2)$$

$$\ln(2x_1 + 3) = \ln(2x_2 + 3)$$

$$2x_1 + 3 = 2x_2 + 3$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

Since  $x_1 = x_2$ , therefore  $f(x)$  is a one-to-one function.

**9(b)**

$$f(x) = \ln(2x + 3), \quad g(x) = \frac{e^x - 3}{2}$$

$$(f \circ g)(x) = f[g(x)]$$

$$= f\left[\frac{e^x - 3}{2}\right]$$

$$= \ln\left[2\left(\frac{e^x - 3}{2}\right) + 3\right]$$

$$= \ln[e^x - 3 + 3]$$

$$= \ln[e^x]$$

$$= x \ln e$$

$$= x$$

$$(g \circ f)(x) = g[f(x)]$$

$$= g[\ln(2x + 3)]$$

$$= \frac{e^{\ln(2x+3)} - 3}{2}$$

$$= \frac{(2x + 3) - 3}{2}$$

$$= x$$

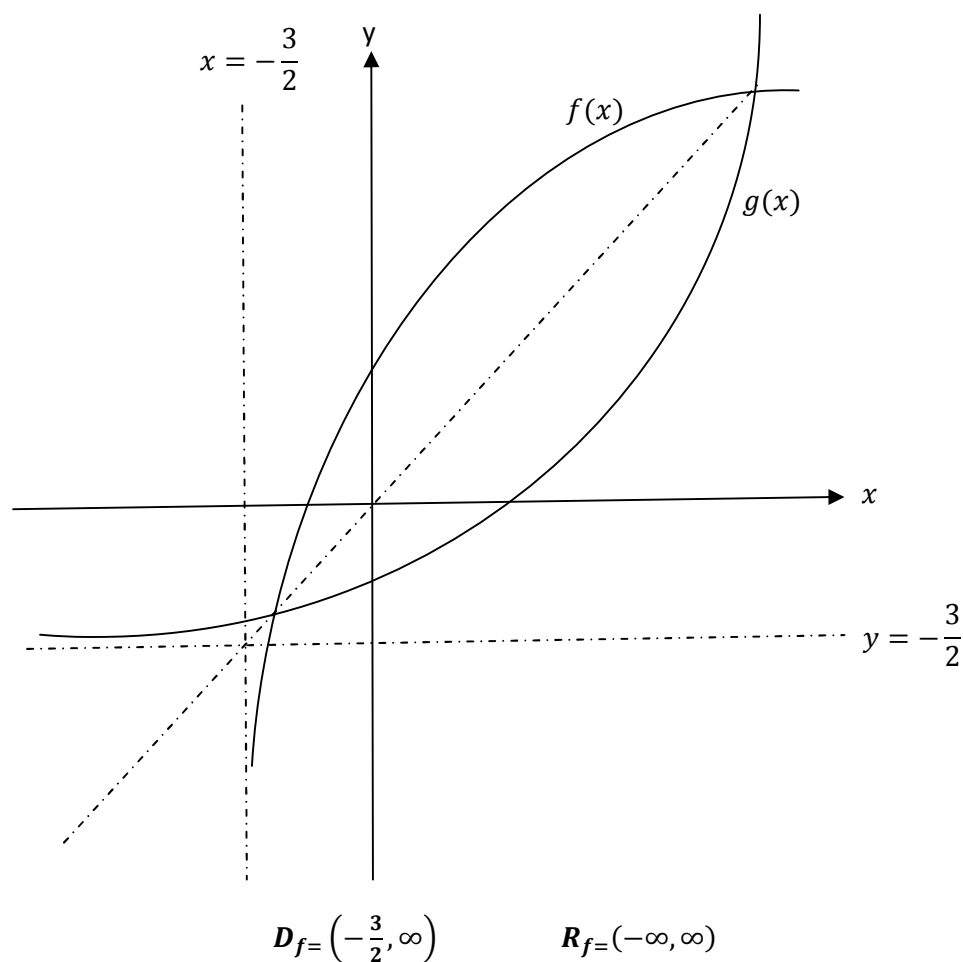
$$e^{\ln x} = x$$

$$e^{\ln(2x+3)} = 2x + 3$$

### Conclusion

Since  $f \circ g(x) = g \circ f(x) = x$ , therefore  $f^{-1}(x) = g(x)$  and  $g^{-1}(x) = f(x)$ .

### 9(c)



10. Given

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix}$$

- (a) Find the determinant of matrix  $A$ .
- (b) Find the minor, cofactor and adjoint of matrix  $A$ .
- (c) Given  $A(\text{adjoint}(A)) = |A|I$  where  $I$  is  $3 \times 3$  identity matrix, show that  $A^{-1} = \frac{1}{|A|}\text{adjoint}(A)$ . Hence, find  $A^{-1}$ .
- (d) By using  $A^{-1}$  in part (c), solve the following simultaneous equations.
- $$\begin{aligned} 2x + 2y + 3z &= 49 \\ x + 5y + 4z &= 74 \\ 3x + y + 4z &= 49 \end{aligned}$$

### SOLUTION

#### 10(a)

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix}$$

$$\begin{aligned} |A| &= (2) \begin{vmatrix} 5 & 4 \\ 1 & 4 \end{vmatrix} - (2) \begin{vmatrix} 1 & 4 \\ 3 & 4 \end{vmatrix} + (3) \begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} \\ &= (2)(20 - 4) - (2)(4 - 12) + (3)(1 - 15) \\ &= (2)(16) - (2)(-8) + (3)(-14) \\ &= 32 + 16 - 42 \\ &= 6 \end{aligned}$$

#### 10(b)

$$\begin{aligned} \text{Minor, } M_{ij} &= \begin{bmatrix} \begin{vmatrix} 5 & 4 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 5 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 1 & 5 \end{vmatrix} \end{bmatrix} \\ &= \begin{bmatrix} (20 - 4) & (4 - 12) & (1 - 15) \\ (8 - 3) & (8 - 9) & (2 - 6) \\ (8 - 15) & (8 - 3) & (10 - 2) \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 16 & -8 & -14 \\ 5 & -1 & -4 \\ -7 & 5 & 8 \end{bmatrix}$$

$$\text{Cofactor, } C_{ij} = \begin{bmatrix} +M_{11} & -M_{12} & +M_{13} \\ -M_{21} & +M_{22} & -M_{23} \\ +M_{31} & -M_{32} & +M_{33} \end{bmatrix}$$

$$= \begin{bmatrix} +(16) & -(-8) & +(-14) \\ -(5) & +(-1) & -(-4) \\ +(-7) & -(5) & +(8) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 8 & -14 \\ -5 & -1 & 4 \\ -7 & -5 & 8 \end{bmatrix}$$

Adjoint A,  $\text{Adj } A = C^T$

$$= \begin{bmatrix} 16 & 8 & -14 \\ -5 & -1 & 4 \\ -7 & -5 & 8 \end{bmatrix}^T$$

$$= \begin{bmatrix} 16 & -5 & -7 \\ 8 & -1 & -5 \\ -14 & 4 & 8 \end{bmatrix}$$

**10(c)**

$$A(\text{adjoint}(A)) = |A|I$$

$$A^{-1}A = I$$

$$A^{-1}A(\text{adjoint}(A)) = A^{-1}|A|I$$

$$I(\text{adjoint}(A)) = A^{-1}|A|I$$

$$(\text{adjoint}(A)) = A^{-1}|A|I$$

$$A^{-1}|A| = (\text{adjoint}(A))$$

$$A^{-1} = \frac{1}{|A|}(\text{adjoint}(A))$$

$$A^{-1} = \frac{1}{6} \begin{bmatrix} 16 & -5 & -7 \\ 8 & -1 & -5 \\ -14 & 4 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{16}{6} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{8}{6} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-14}{6} & \frac{4}{6} & \frac{8}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{3} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{4}{3} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-7}{3} & \frac{2}{3} & \frac{4}{3} \end{bmatrix}$$

**10(d)**

$$2x + 2y + 3z = 49$$

$$x + 5y + 4z = 74$$

$$3x + y + 4z = 49$$

$$\begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 49 \\ 74 \\ 49 \end{bmatrix}$$

$$\begin{bmatrix} \frac{8}{3} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{4}{3} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-7}{3} & \frac{2}{3} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} 2 & 2 & 3 \\ 1 & 5 & 4 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{8}{3} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{4}{3} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-7}{3} & \frac{2}{3} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} 49 \\ 74 \\ 49 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{8}{3} & \frac{-5}{6} & \frac{-7}{6} \\ \frac{4}{3} & \frac{-1}{6} & \frac{-5}{6} \\ \frac{-7}{3} & \frac{2}{3} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} 49 \\ 74 \\ 49 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{71}{6} \\ \frac{73}{6} \\ \frac{1}{3} \end{bmatrix}$$

$$\therefore x = \frac{71}{6}, \quad y = \frac{73}{6}, \quad z = \frac{1}{3}$$

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$