QS015/2
Mathematics
Paper 2
Semester I
Session 2013/2014
2 hours

QS015/2
Matematik
Kertas 2
Semester I
Sesi 2013/2014
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PENDIDIKAN MALAYSIA
MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **19** halaman bercetak.
This question paper consists of **19** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin\ (A \pm B) = \sin A \cos B \ \pm\ \cos A \sin B$$

$$\cos\ (A \pm B) = \cos A \cos B \ \mp\ \sin A \sin B$$

$$\tan\ (A \pm B) = \frac{\tan A \ \pm\ \tan B}{1 \ \mp\ \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin\ 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos\ 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

$$\tan\ 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$\sin^2 A = \frac{1-\cos 2A}{2}$$

$$\cos^2 A = \frac{1+\cos 2A}{2}$$

LIST OF MATHEMATICAL FORMULAE

Differentiation

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

If $x = f(t)$ and $y = g(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Sphere	$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$
Right circular cone	$V = \frac{1}{3}\pi r^2 h$	$S = \pi rs$
Right circular cylinder	$V = \pi r^2 h$	$S = 2\pi rh$

1 Express $\dfrac{x^2}{x^2+3x+2}$ in partial fractions form.

[5 *marks*]

2 State the values of R and α such that $3\sin\theta+6\cos\theta = R\sin(\theta+\alpha)$ where $R>0$ and $0^\circ<\alpha\leq 90^\circ$. Hence, solve $3\sin\theta+6\cos\theta=\sqrt{5}$ for $0^\circ\leq\theta<180^\circ$.

[6 *marks*]

3 (a) Find the value of m if $\lim\limits_{x\to 0}\dfrac{mx+3x^2}{4x-8x^2}=3$.

[3 *marks*]

(b) Evaluate $\lim\limits_{x\to 0}\dfrac{\sqrt{3-x}-\sqrt{3}}{x}$.

[4 *marks*]

4 (a) Find $\dfrac{dy}{dx}$ if $y=\operatorname{cosec}\left\{\sin\left[\ln\left(x+1\right)\right]\right\}$.

[3 *marks*]

(b) Obtain the second derivative of $y=\dfrac{\cos 3x}{e^{2x}}$ and express your answer in the simplest form.

[4 *marks*]

Chow Choon Wooi

5 A cubic polynomial $P(x)$ has remainders 3 and 1 when divided by $(x-1)$ and $(x-2)$, respectively.

(a) Let $Q(x)$ be a linear factor such that $P(x)=(x-1)(x-2)Q(x)+\alpha x+\beta$, where α and β are constants. Find the remainder when $P(x)$ is divided by $(x-1)(x-2)$.

[5 *marks*]

(b) Use the values of α and β from part (a) to determine $Q(x)$ if the coefficient of x^3 for $P(x)$ is 1 and $P(3)=7$. Hence, solve for x if $P(x)=7-3x$.

[6 *marks*]

6 (a) State the definition of the continuity of a function at a point. Hence, find the value of d such that

$$f(x)=\begin{cases} e^{3x+d}, & x\le 0 \\ 3x+5, & x>0 \end{cases}$$

is continuous at $x=0$.

[5 *marks*]

(b) A function f is defined by

$$f(x)=\begin{cases} x^2-1, & x\le 1 \\ k(x-1), & x>1. \end{cases}$$

Determine the value(s) of k if f is:

(i) continuous for all $x\in\mathbb{R}$.

[3 *marks*]

(ii) differentiable for all $x\in\mathbb{R}$.

[4 *marks*]

7 (a) Find the derivative of $f(x)=\dfrac{1}{x+1}$ by using the first principle.

[4 *marks*]

(b) Use implicit differentiation to find:

(i) $\dfrac{dy}{dx}$ if $y\ln x=e^{x-y}$.

[3 *marks*]

(ii) the value of $\dfrac{dy}{dx}$ if $\dfrac{1}{y}-\dfrac{1}{x}=3$ when $x=\dfrac{1}{2}$.

[5 *marks*]

8 A curve is defined by parametric equations

$$x=\ln(1+t),\ \ y=e^{t^2}\ \text{ for } t>-1.$$

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ in terms of t.

[6 *marks*]

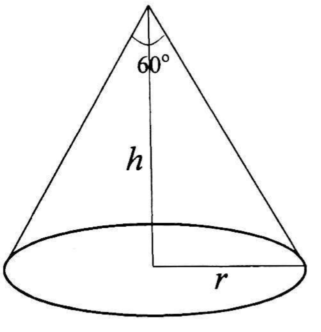
(b) Show that the curve has only one relative extremum at $(0,1)$ and determine the nature of the point.

[6 *marks*]

9 (a) A cylindrical container of volume $128\pi \text{ m}^3$ is to be constructed with the same material for the top, bottom and lateral side. Find the dimensions of the container that will minimise the amount of the material needed.

[6 *marks*]

(b) Gravel is poured onto a flat ground at the rate of $\frac{3}{20} \text{ m}^3$ per minute to form a conical-shaped pile with vertex angle $60°$ as shown in the diagram below.



Compute the rate of change of the height of the conical pile at the instant $t = 10$ minutes.

[7 *marks*]

10 (a) Show that $\dfrac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \cot\left(\dfrac{\beta - \alpha}{2}\right)$.

[4 *marks*]

(b) Use trigonometric identities to verify that

(i) $\sin\theta = \dfrac{2\tan\dfrac{\theta}{2}}{1 + \tan^2\dfrac{\theta}{2}}$.

[3 *marks*]

(ii) $\cos\theta = \dfrac{1 - \tan^2\dfrac{\theta}{2}}{1 + \tan^2\dfrac{\theta}{2}}$.

[3 *marks*]

Hence, solve the equation $3\sin\theta + \cos\theta = 2$ for $0^\circ \le \theta \le 180^\circ$. Give your answers correct to three decimal places.

[5 *marks*]

END OF QUESTION PAPER