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QM 016

Mid-Semester Examination

Semester I

Session 2009/2010

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1. (a) Solve $9^{4x+1} = 27$.

   (b) Given $z_1 = 4 - 3i$ and $z_2 = 3 + 2i$. Find $z_1 z_2$ in the form of a+bi.

2. If α and β are roots of the equation $5x^2 - 2x - 9 = 0$, find the quadratic equation whose roots are α^2 and β^2. **[Out of QS015 Syllabus]**

3. Solve $|x + 4| > 2|x - 1|$.

4. The polynomial $P(x) = 2x^3 - 3ax^2 + ax + b$ has (x -1) as a factor and leaves a remainder of -54 when divided by (x + 2). Find the values of a and b.
   [QS015 Chapter 6]

5. Express $\frac{3}{(x-1)^2(x+1)}$ as partial fractions. **[QS015 Chapter 6]**

6. Given that $log_2 x - log_x 8 + log_2 2^k + k\, log_x 4 = 0$. If y = $log_2 x$, show that $y^2 + ky + 2k - 3 = 0$. Find

   (a) the set of values of k for which y is real.

   (b) the value of x when k = 6.

7. Show that $(1 + ax)^{-\frac{1}{2}} = 1 - \frac{a}{2}x + \frac{3a^2}{8}x^2 + \ ...$

   Given that $(1 + x^2)(1 + ax)^{-\frac{1}{2}} = 1 + x + bx^2 +$ .... Find the values of a and b. State the set of values of x for which the above expansion is valid.

1. (a) Solve $9^{4x+1} = 27$.

(b) Given $z_1 = 4 - 3i$ and $z_2 = 3 + 2i$. Find $z_1 z_2$ in the form of a+bi.

Solution

(a) $9^{4x+1} = 27$

$3^{2(4x+1)} = 3^3$

$2(4x + 1) = 3$

$8x + 2 = 3$

$8x = 1$

$x = \frac{1}{8}$

(b) $z_1 z_2$

$= (4 - 3i)(3 + 2i)$

$= 12 - 9i + 8i - 6i^2$

$= 12 - 9i + 8i + 6$

$= 18 - i$

2. If α and β are roots of the equation $5x^2 - 2x - 9 = 0$, find the quadratic equation whose roots are α^2 and β^2. **[Out of QS015 Syllabus]**

Solution

$$5x^2 - 2x - 9 = 0$$

$$x^2 - \frac{2}{5}x - \frac{9}{5} = 0$$

Sum of roots $\alpha + \beta = \frac{2}{5}$

Product of roots $\alpha\beta = -\frac{9}{5}$

$$\alpha^2 + \beta^2$$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{2}{5}\right)^2 - 2\left(-\frac{9}{5}\right)$$

$$= \frac{4}{25} + \frac{18}{5}$$

$$= \frac{94}{25}$$

$$\alpha^2\beta^2$$

$$= (\alpha\beta)^2$$

$$= \left(-\frac{9}{5}\right)^2$$

$$= \frac{81}{25}$$

The new quadratic equation is

$$x^2 - (sum\ of\ roots)x + (product\ of\ roots) = 0$$

$$x^2 - \left(\frac{94}{25}\right)x + \left(\frac{81}{25}\right) = 0$$

$$25x^2 - 94x + 81 = 0$$

3. Solve $|x+4| > 2|x-1|$.

Solution

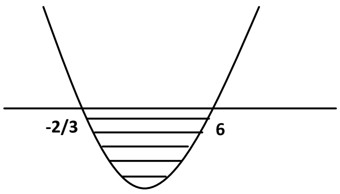
$|x+4| > 2|x-1|$

$(x+4)^2 > [2(x-1)]^2$

$(x^2 + 8x + 16) > (4x^2 - 8x + 4)$

$3x^2 - 16x - 12 < 0$

$(3x+2)(x-6) < 0$



Solution set: $\left\{x: \frac{2}{3} < x < 6\right\}$

4. The polynomial $P(x) = 2x^3 - 3ax^2 + ax + b$ has (x -1) as a factor and leaves a remainder of -54 when divided by (x + 2). Find the values of a and b.
[QS015 Chapter 6]

Solution

$P(x) = 2x^3 - 3\mathrm{a}\mathrm{x}^2 + \mathrm{a}\mathrm{x} + \mathrm{b}$

$P(1) = 2(1)^3 - 3\mathrm{a}(1)^2 + \mathrm{a}(1) + \mathrm{b} = 0$

$2 - 3\mathrm{a} + \mathrm{a} + \mathrm{b} = 0$

$2a - \mathrm{b} = 2$ .... (1)

$P(-2) = 2(-2)^3 - 3\mathrm{a}(-2)^2 + \mathrm{a}(-2) + \mathrm{b} = -54$

$2(-8) - 3\mathrm{a}(4) - 2\mathrm{a} + \mathrm{b} = -54$

$14a - b = 38$ .... (2)

(2)– (1): 12a = 36

a = 3

b = 4

$\therefore a = 3, \quad b = 4$

5. Express $\frac{3}{(x-1)^2(x+1)}$ as partial fractions. **[QS015 Chapter 6]**

Solution

$$\frac{3}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$$3 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$When\ x = 1,$

$$3 = 2B$$

$$B = \frac{3}{2}$$

$When\ x = -1$

$$3 = 4C$$

$$C = \frac{3}{4}$$

$When\ x = 0$

$$3 = -A + \frac{3}{2} + \frac{3}{4}$$

$$A = -\frac{3}{4}$$

$$\therefore A = -\frac{3}{4},\ B = \frac{3}{2},\ C = \frac{3}{4}$$

$$\frac{3}{(x-1)^2(x+1)} = -\frac{3}{4(x-1)} + \frac{3}{2(x-1)^2} + \frac{3}{4(x+1)}$$

6. Given that $log_2 x - log_x 8 + log_2 2^k + k\, log_x 4 = 0$. If y = $log_2 x$, show that $y^2 + ky + 2k - 3 = 0$. Find

(a) the set of values of k for which y is real.

(b) the value of x when k = 6.

Solution

$$log_2 x - log_x 8 + log_2 2^k + k\, log_x 4 = 0$$

$$log_2 x - \frac{log_2 8}{log_2 x} + k log_2 2 + \frac{k\, log_2 4}{log_2 x} = 0$$

Let $y = log_2 x$

$$y - \frac{log_2 2^3}{y} + k(1) + \frac{k\, log_2 2^2}{y} = 0$$

$$y - \frac{3 log_2 2}{y} + k(1) + \frac{2k\, log_2 2}{y} = 0$$

$$y - \frac{3(1)}{y} + k(1) + \frac{2k(1)}{y} = 0$$

$$y - \frac{3}{y} + k + \frac{2k}{y} = 0$$

$$y^2 - 3 + ky + 2k = 0$$

$$y^2 + ky + (2k - 3) = 0$$

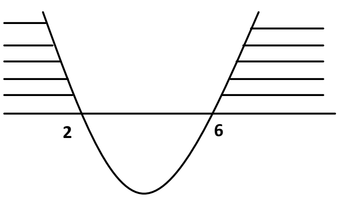
(a) For real y:

$$b^2 - 4ac \geq 0$$

$$(k)^2 - 4(1)(2k - 3) \geq 0$$

$$k^2 - 8k + 12 \geq 0$$

$$(k - 2)(k - 6) \geq 0$$



The set of values of k is $\{k : k \leq 2 \text{ or } k \geq 6\}$

(c) When k = 6,

$$y^2 + 6y + 9 = 0$$

$$(y+3)^2 = 0$$

$$y + 3 = 0$$

$$y = -3$$

When y = -3

$$log_2 x = -3$$

$$x = 2^{-3} = \frac{1}{8}$$

7. Show that $(1+ax)^{-\frac{1}{2}} = 1 - \frac{a}{2}x + \frac{3a^2}{8}x^2 + \ ...$

Given that $(1+x^2)(1+ax)^{-\frac{1}{2}} = 1 + x + bx^2 + \ ....$ Find the values of a and b. State the set of values of x for which the above expansion is valid.

Solution

$$(1+ax)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(ax) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(ax)^2 + \ ...$$

$$= 1 - \frac{a}{2}x + \frac{3a^2}{8}x^2 + \ ...$$

$$(1+x^2)(1+ax)^{-\frac{1}{2}} = (1+x^2)\left(1 - \frac{a}{2}x + \frac{3a^2}{8}x^2 + \ ...\right)$$

$$= 1 - \frac{a}{2}x + \frac{3a^2}{8}x^2 + x^2 \ ...$$

$$= 1 - \frac{a}{2}x + \left(\frac{3a^2}{8} + 1\right)x^2 \ ...$$

$$= 1 + x + bx^2 + \ ...$$

$-\frac{a}{2} = 1$ ➔ a = -2

$b = \frac{3a^2}{8} + 1$ ➔ b = $\frac{5}{2}$

$|-2x| < 1$

$-\frac{1}{2} < x < \frac{1}{2}$