QS 015

Mid-Semester Examination

Semester I

Session 2011/2012

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1. Given $Z_1 = 1 + 2i$ and $Z_2 = 3 - 2i$. Express $Z_1 + \frac{1}{\bar{Z}_2}$ in the form of $a + bi$ where $\bar{Z}_2$ is the conjugate of Z_2.

2. Solve $4^x - 3(2^x) - 4 = 0$.

3. (a) Find the determinant of $A = \begin{bmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{bmatrix}$.

   (b) Given $a = log_x 2$ and $b = log_x 3$, express $log_x\left(\frac{9\sqrt{x}}{4}\right)$ in terms of a and b.

4. Solve the inequality $\left|\frac{3-2x}{x-1}\right| > 3$.

5. (a) The sum of first n terms of an arithmetic progression is $S_n = 2n^2 + 7n$. Find the first term and the commond difference. Hence, find the 8th term of the progression.

   (b) Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x until the term including x^3. By substituting $x = \frac{1}{50}$, evalute $\sqrt{26}$ correct to four decimal places.

6. Given $A = \begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix}$.

   (a) Show that AB = 6I.

   (b) Deduce A^{-1}.

   (c) Hence, solve the following system of linear equations.

   -4x + 2y + 2z = 1
   6x – 4y + 2z = 2
   -2x + 4y -2x = 14

1. Given $Z_1 = 1 + 2i$ and $Z_2 = 3 - 2i$. Express $Z_1 + \frac{1}{\bar{Z}_2}$ in the form of $a + bi$ where $\bar{Z}_2$ is the conjugate of Z_2.

Solution

$$Z_2 = 3 - 2i$$

$$Z_1 + \frac{1}{\bar{Z}_2}$$

$$= 1 + 2i + \frac{1}{3 + 2i}$$

$$= 1 + 2i + \frac{1}{3 + 2i}\left(\frac{3 - 2i}{3 - 2i}\right)$$

$$= 1 + 2i + \frac{3 - 2i}{13}$$

$$= \frac{13 + 26i + 3 - 2i}{13}$$

$$= \frac{16}{13} + \frac{24}{13}i$$

2. Solve $4^x - 3(2^x) - 4 = 0$.

Solution

$4^x - 3(2^x) - 4 = 0$

$2^{2x} - 3(2^x) - 4 = 0$

Let $y = 2^x$

$y^2 - 3y - 4 = 0$

$(y-4)(y+1) = 0$

$y = 4 \quad or \quad y = -1$

$2^x = 4 \qquad\qquad 2^x = -1$(Undefined)

$x = 2$

3. (a) Find the determinant of $A = \begin{bmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{bmatrix}$.

(b) Given $a = log_x 2$ and $b = log_x 3$, express $log_x \left(\frac{9\sqrt{x}}{4}\right)$ in terms of a and b.

Solution

(a) $|A| = (2)\begin{vmatrix} 2 & -3 \\ -6 & 2 \end{vmatrix} - (3)\begin{vmatrix} 6 & -3 \\ 3 & 2 \end{vmatrix} + (6)\begin{vmatrix} 6 & 2 \\ 3 & -6 \end{vmatrix}$

$= 2(4 - 18) - 3(12 + 9) + 6(-36 - 6)$

$= -343$

(b) $log_x \left(\frac{9\sqrt{x}}{4}\right)$

$= log_x 9 + log_x \sqrt{x} - log_x 4$

$= log_x 3^2 + log_x x^{\frac{1}{2}} - log_x 2^2$

$= 2log_x 3 + \frac{1}{2} log_x x - 2log_x 2$

$= 2b + \frac{1}{2} - 2a$

4. Solve the inequality $\left|\frac{3-2x}{x-1}\right| > 3$

Solution

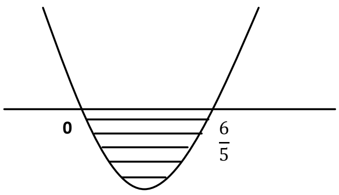
$$\left|\frac{3-2x}{x-1}\right| > 3$$

$$(3-2x)^2 > 9(x-1)^2$$

$$9 - 12x + 4x^2 > 9(x^2 - 2x + 1)$$

$$5x^2 - 6x < 0$$

$$x(5x-6) < 0$$



Therefore the solution is $(0, 1) \cup \left(1, \frac{6}{5}\right)$

5. (a) The sum of first n terms of an arithmetic progression is $S_n = 2n^2 + 7n$. Find the first term and the commond difference. Hence, find the 8th term of the progression.

(b) Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x until the term including x^3. By substituting $x = \frac{1}{50}$, evalute $\sqrt{26}$ correct to four decimal places.

Solution

(a) $a = S_1 = 2(1)^2 + 7(1)$

$a = S_1 = 9$

$S_2 = 2(2)^2 + 7(2) = 22$

$T_2 = S_2 - S_1 = 22 - 9 = 13$

$d = T_2 - a = 13 - 9 = 4$

$T_8 = 9 + 7(4)$

$T_8 = 37$

(b) $(1 + 2x)^{\frac{1}{2}}$

$$= 1 + \frac{1}{2}(2x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2} - 1\right)}{2!}(2x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{3!}(2x)^3 + \ldots$$

$$= 1 + x + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(4x^2) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}(8x^3) + \ldots$$

$$= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \ldots$$

When $x = \frac{1}{50}$

$$\sqrt{1 + 2\left(\frac{1}{50}\right)} = 1 + \left(\frac{1}{50}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{50}\right)^2 + \left(\frac{1}{2}\right)\left(\frac{1}{50}\right)^3 + \ldots$$

$$\sqrt{\frac{26}{25}} = 1.019804$$

$$\frac{\sqrt{26}}{5} = 1.019804$$

$$\sqrt{26} = 5\,(1.019804)$$

$$\sqrt{26} = 5.0990$$

6. Given $A = \begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix}$.

(a) Show that AB = 6I.

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(c) Hence, solve the following system of linear equations.

-4x + 2y + 2z = 1
6x – 4y + 2z = 2
-2x + 4y -2x = 14

Solution

(a) $AB = \begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$= 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= 6I$

(b) $A^{-1} = \frac{1}{6}B$

$= \frac{1}{6}\begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{5}{6} \\ \frac{2}{3} & \frac{1}{2} & \frac{1}{6} \end{bmatrix}$

(c) -4x + 2y + 2z = 1 .... (1)
6x – 4y + 2z = 2 .... (2)
-2x + 4y -2x = 14 .... (3)

$$\begin{bmatrix} -4 & 2 & 2 \\ 6 & -4 & 2 \\ -2 & 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 14 \end{bmatrix}$$

$$2\begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 2 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 1 \\ 7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} \\ 1 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ \frac{13}{2} \\ 2 \end{bmatrix}$$

$$\therefore x = 4, \quad y = \frac{13}{2}, \quad z = 2$$